The significance and interpretation of recovery data

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Last update: April 28, 2025

1. Notes

2. Additional readings

- van der Kamp (1989)
- Neville and van der Kamp (2009)
- Neville and van der Kamp (2012)

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Overview

Monitoring the recovery after a pumping test is generally mandated; however, in our experience we have generally found that little attention is paid to the data that are collected. Recovery data frequently provide some of the most reliable information from pumping tests and may provide important insights into aquifer response that cannot be obtained from the pumping phase of the test. These notes have been prepared to highlight the importance of recovery data and to review their interpretation. The final section of the notes is devoted to the use of recovery data to extend the effective duration of pumping.

Outline

1. The significance of recovery data
2. The “smoothing effect” of recovery
3. Interpretation of recovery data: The principle of superposition
4. Cooper and Jacob (1946) straight-line analysis of recovery data
5. Insights into aquifer response from recovery data
6. The van der Kamp (1989) method to extend the effective duration of pumping
7. Case study: Estevan, Saskatchewan
8. Key points
9. References

1. The significance of recovery data

Guidance documents for conducting pumping tests typically require that water levels be monitored for a specified time following the end of pumping. In our experience, frequently nothing is done with the recovery data after they have been collected, plotted, and included in the appendix to a report. In some cases, the cursory treatment of recovery data represents a genuine loss. In our opinion, the significance of recovery data is frequently overlooked. Recovery data frequently provide some of the most reliable information from pumping tests.

There are at least five reasons to consider recovery data.

- In some cases, the only data available are recovery data.
- Recovery data provide a useful check on the reliability of the drawdown observations and on the interpretations drawn from the drawdown data.
- Recovery measurements are largely free of the noise caused by small variations in the pumping rate at the early stages of a pumping test.
- Recovery measurements can be used to assess whether there was a background trend in water levels during a pumping test.
- Recovery data may help diagnose the presence of hydrologic boundaries.

2. The “smoothing effect” of recovery

An important feature of recovery data is that they are largely free of the noise that arises from irregularities in the pumping rate at early time. At the beginning of a constant-rate pumping test it is common to throttle the pump valve to achieve a constant rate. Two hypothetical examples are presented to illustrate how variations in the pumping rate are smoothed during recovery.

Example 1

A pumping test is conducted in an ideal aquifer that is 5 m thick, with a horizontal hydraulic conductivity of 10^{-5} m/sec and a specific storage of 10^{-5} m^{-1}. The aquifer is pumped for just over 1 day and the drawdowns are monitored at a distance of 5 m from the pumping well. The pumping rate is held at a constant rate of about 10 USgpm, except for a brief period of adjustment during the first 10 minutes.

The pumping history is tabulated below.

Time (minutes)	**Pumping rate (USgpm)**	**Pumping rate (m^3/sec)**
0-5	19	1.210E-3
5-10	14	9.075E-4
10→	10	6.309E-4

The pumping history is also plotted in Figure 1. The deviations from the average pumping rate are brief. If this were an actual test, it is likely they would be missed altogether unless the pumping rate was being recorded continuously.

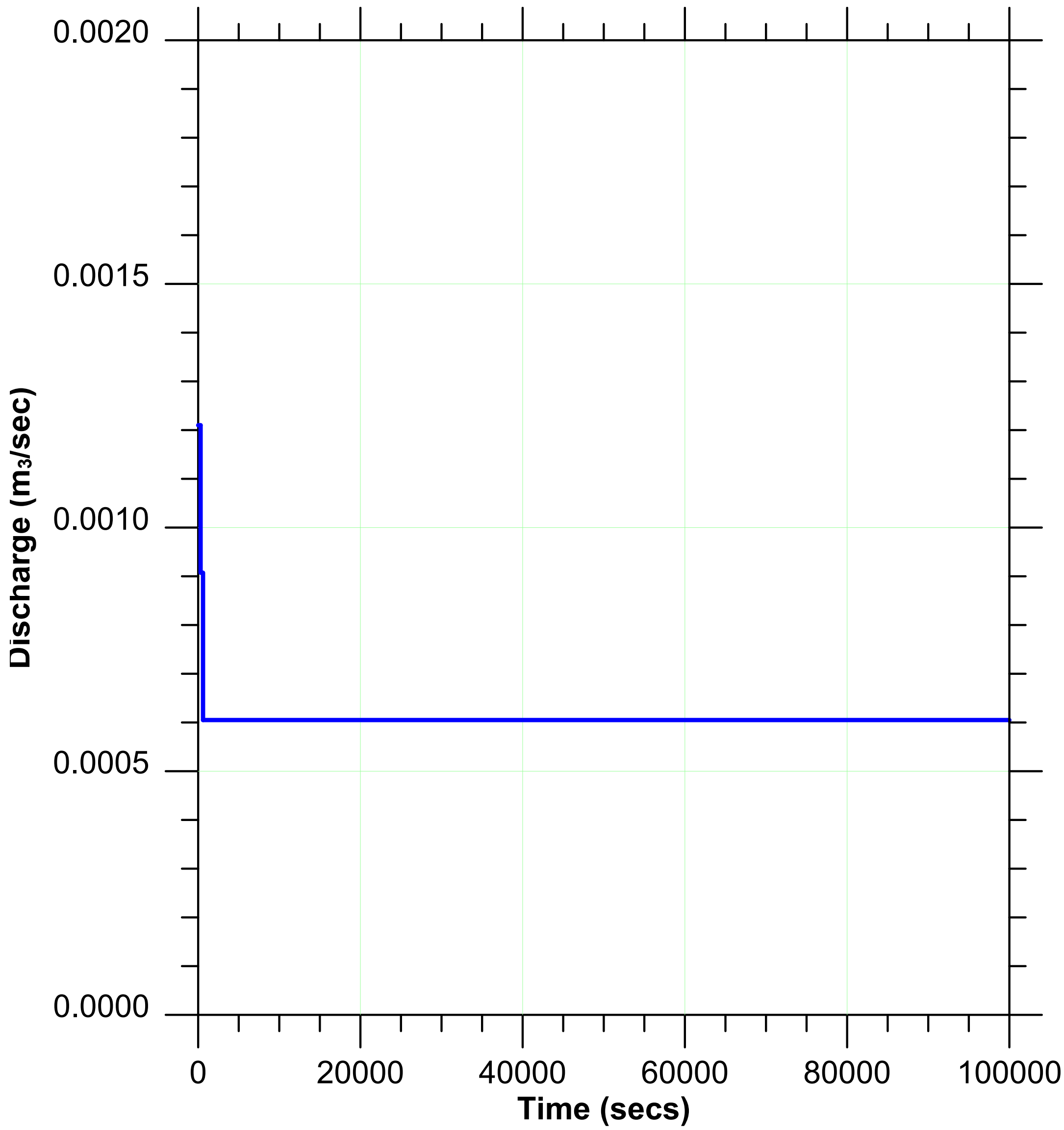


Figure 1. Pumping history for Example 1

The calculated drawdowns for an observation well 5 m from the pumping well are plotted in Figure 2. Because we have plotted the data on a semi-logarithmic time axis, we see that the variations in the pumping rate at the very start of the test appear to have a significant effect on the observed drawdowns. If we did not notice that the pumping rate had varied, we might not immediately recognize that the appropriate portion of the response over which the transmissivity should be estimated is from about 1,000 seconds onwards.

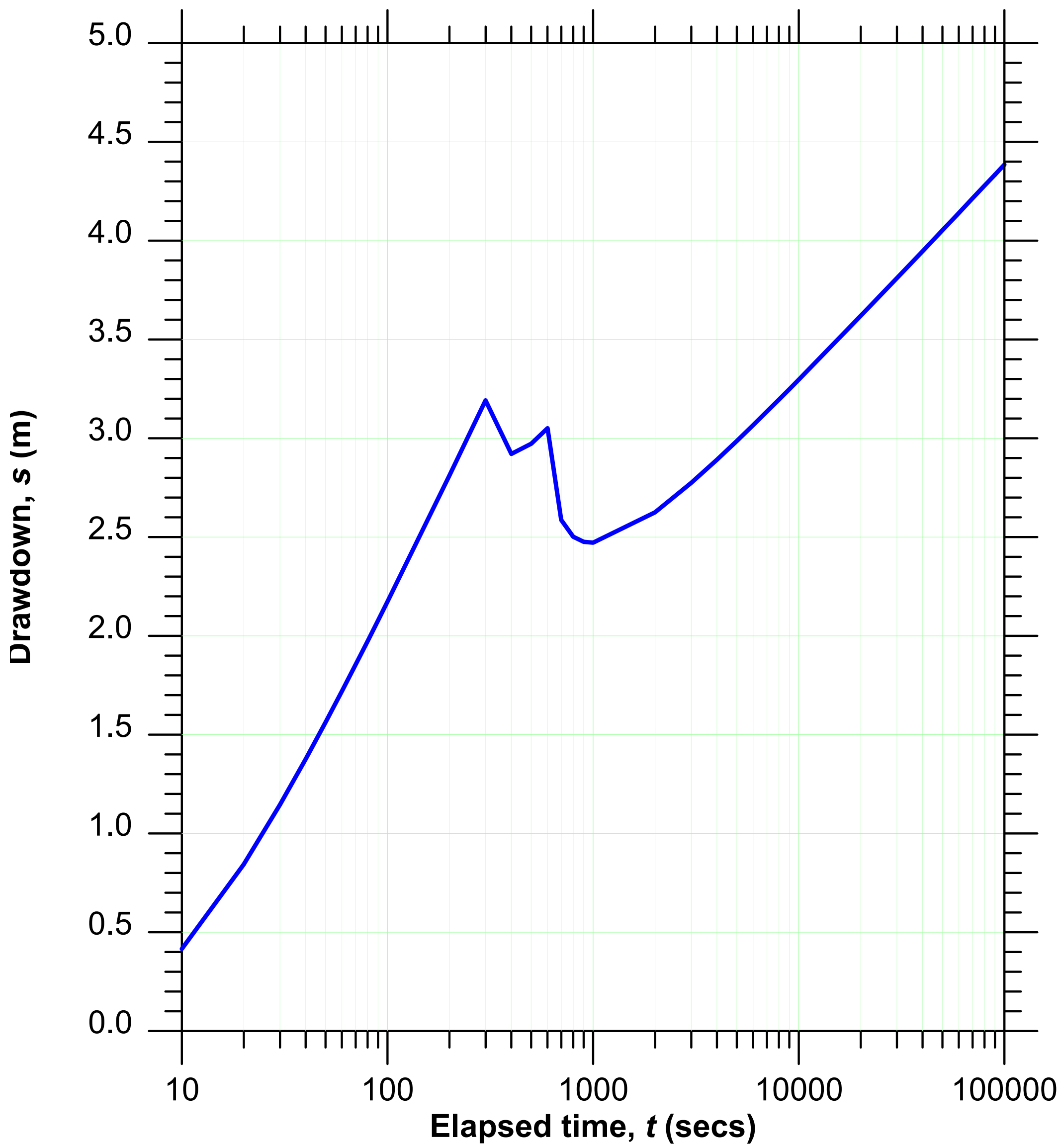


Figure 2. Drawdowns calculated at *r* = 5 m

The complete time-drawdown record is presented in Figure 3. As shown in the figure, the fluctuations in the pumping rate at the start of the test have no effect on the drawdowns recorded following the end of pumping.

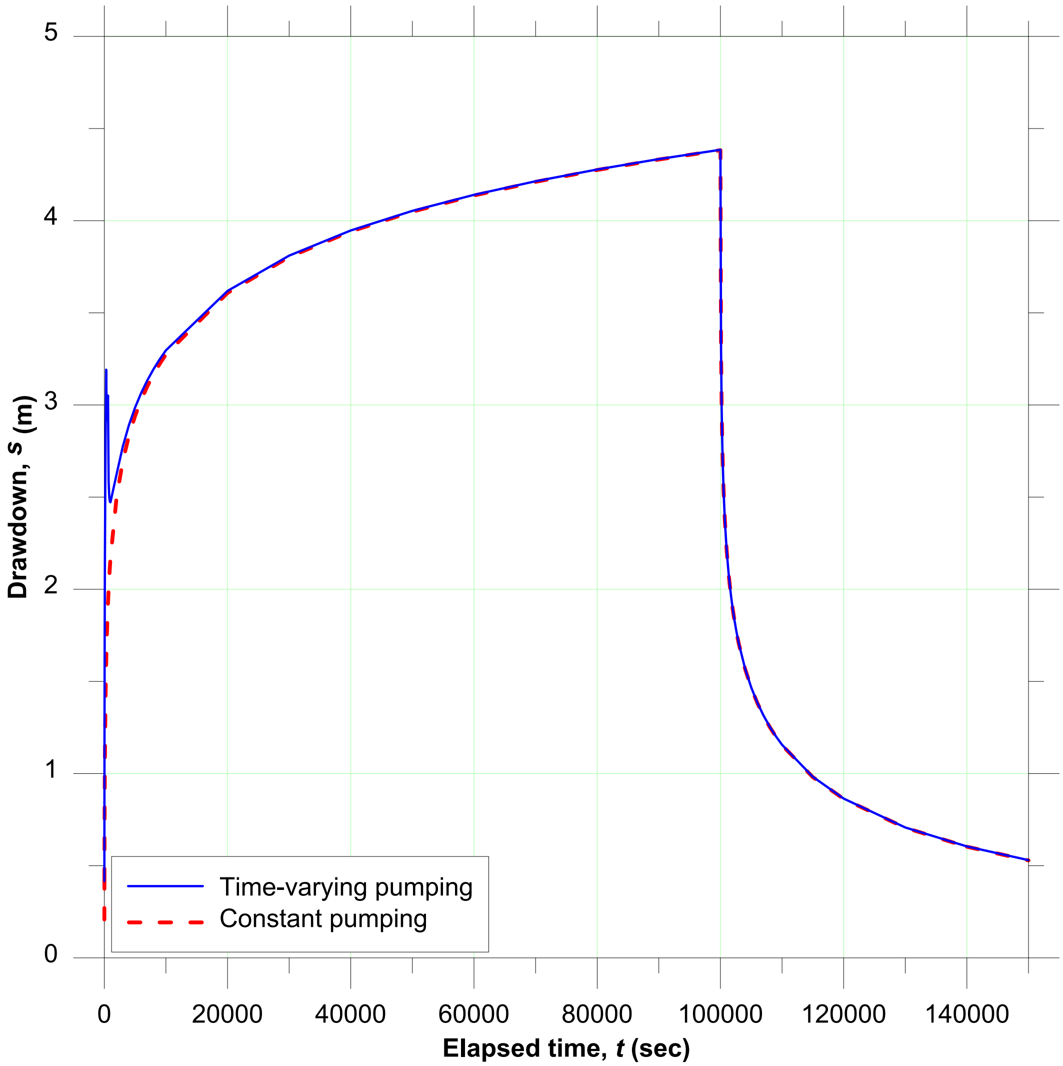


Figure 3. Complete drawdown record for the example

The drawdown data following the end of pumping are shown in Figure 4. Two sets of calculated drawdowns are plotted. The first set corresponds to the drawdowns with the specified time-varying pumping history. The second set corresponds to the drawdowns that would have been observed if the pumping rate had remained constant at the rate that was maintained beyond the first 10 minutes of the test. Consistent with Figure 3, the drawdowns are almost identical; the early variations in the pumping rate do not influence the recovery portion of the response.

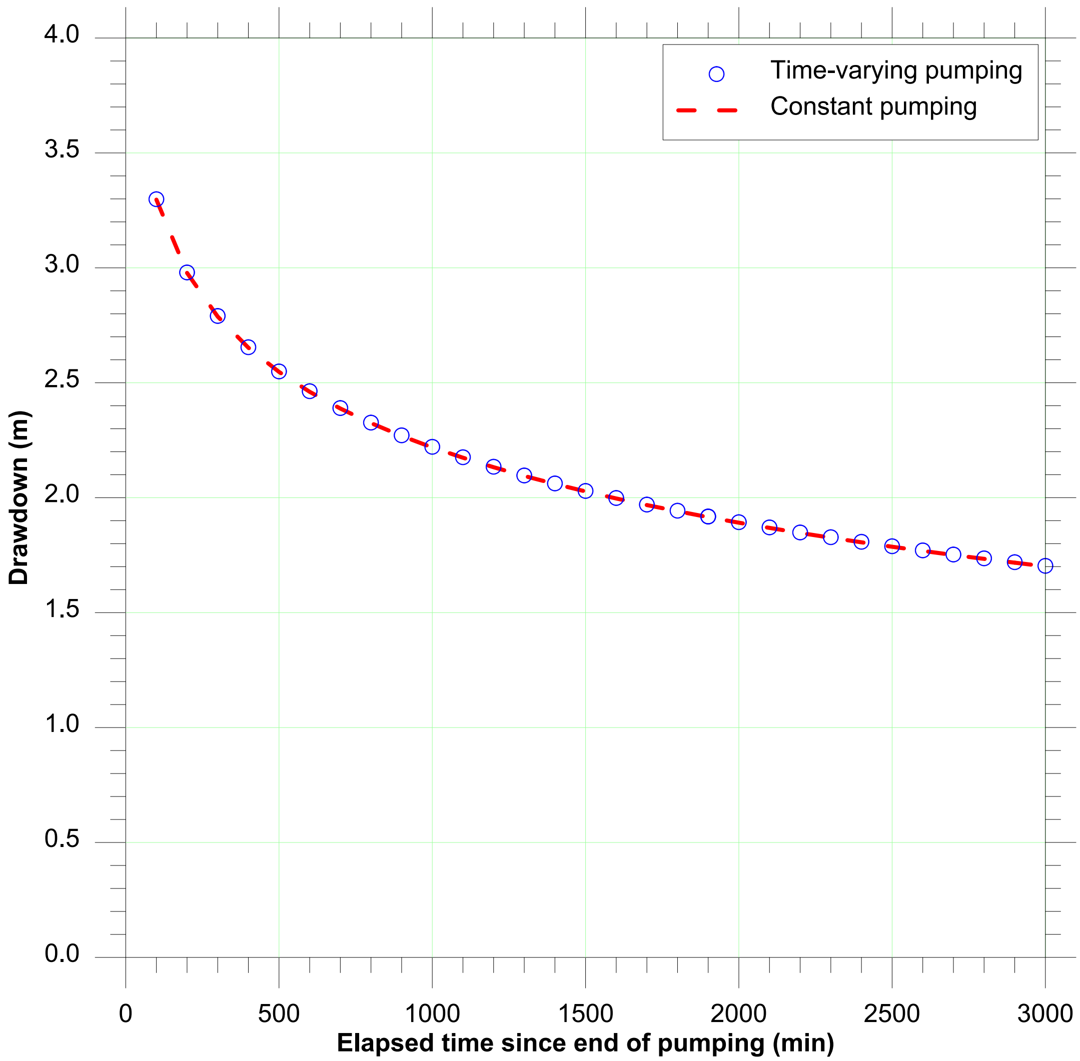


Figure 4. Drawdowns at *r* = 5 m following the end of pumping

Example 2

We use another hypothetical example to show that the smoothing during recovery occurs even when there are significant variations in the pumping rate throughout a test. The pumping rate for the second example is plotted in Figure 5.

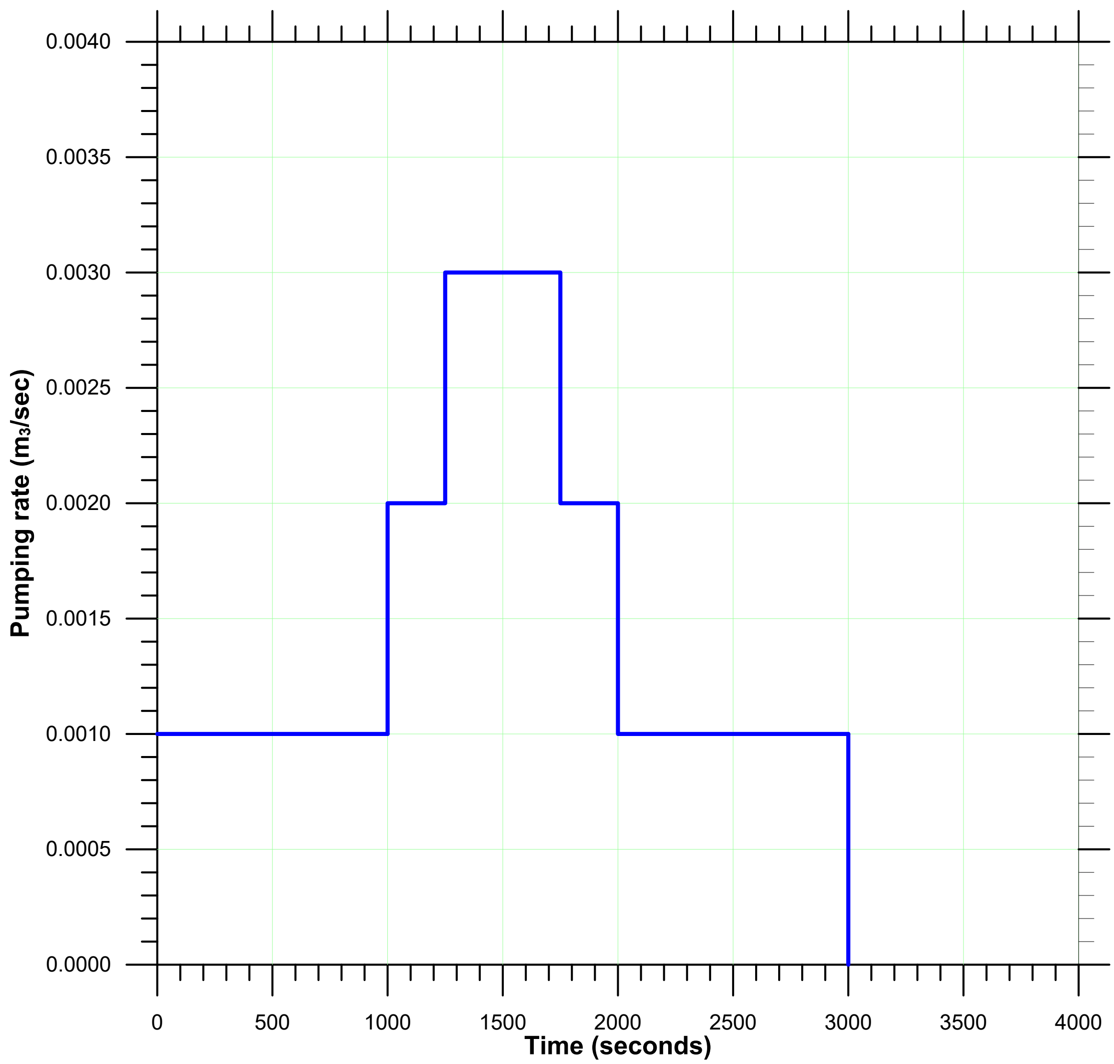


Figure 5. Pumping history for Example 2

The drawdowns calculated for the pumping well are shown in Figure 6. The solid line indicates the drawdowns for time-varying pumping. The dashed line indicates the drawdowns that would have been observed if the pumping rate had remained constant throughout at the average rate. The average rate is defined as the rate that yields the same cumulative volume at the end of pumping. For this example, the cumulative volume pumped over 3,000 seconds is 4.5 m^3, so the average pumping rate is 0.0015 m^3/sec.

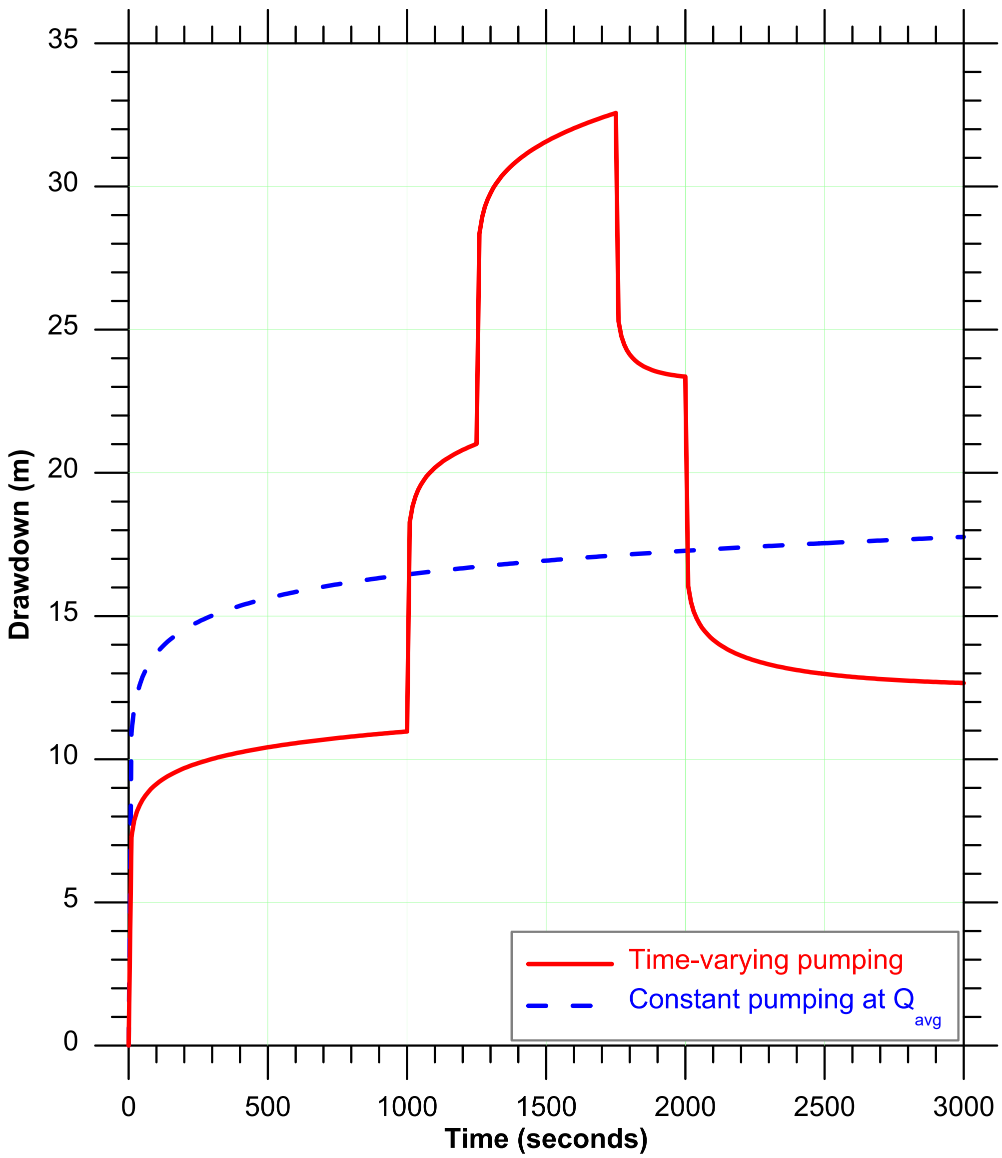


Figure 6. Comparison of drawdowns for time-varying and constant pumping

The drawdowns and subsequent recovery are plotted for both cases in Figure 7. The solid red line indicates the calculated response for time-varying pumping and the dashed blue line indicates the response for constant pumping. The similarity of the water levels after the cessation of pumping is striking.

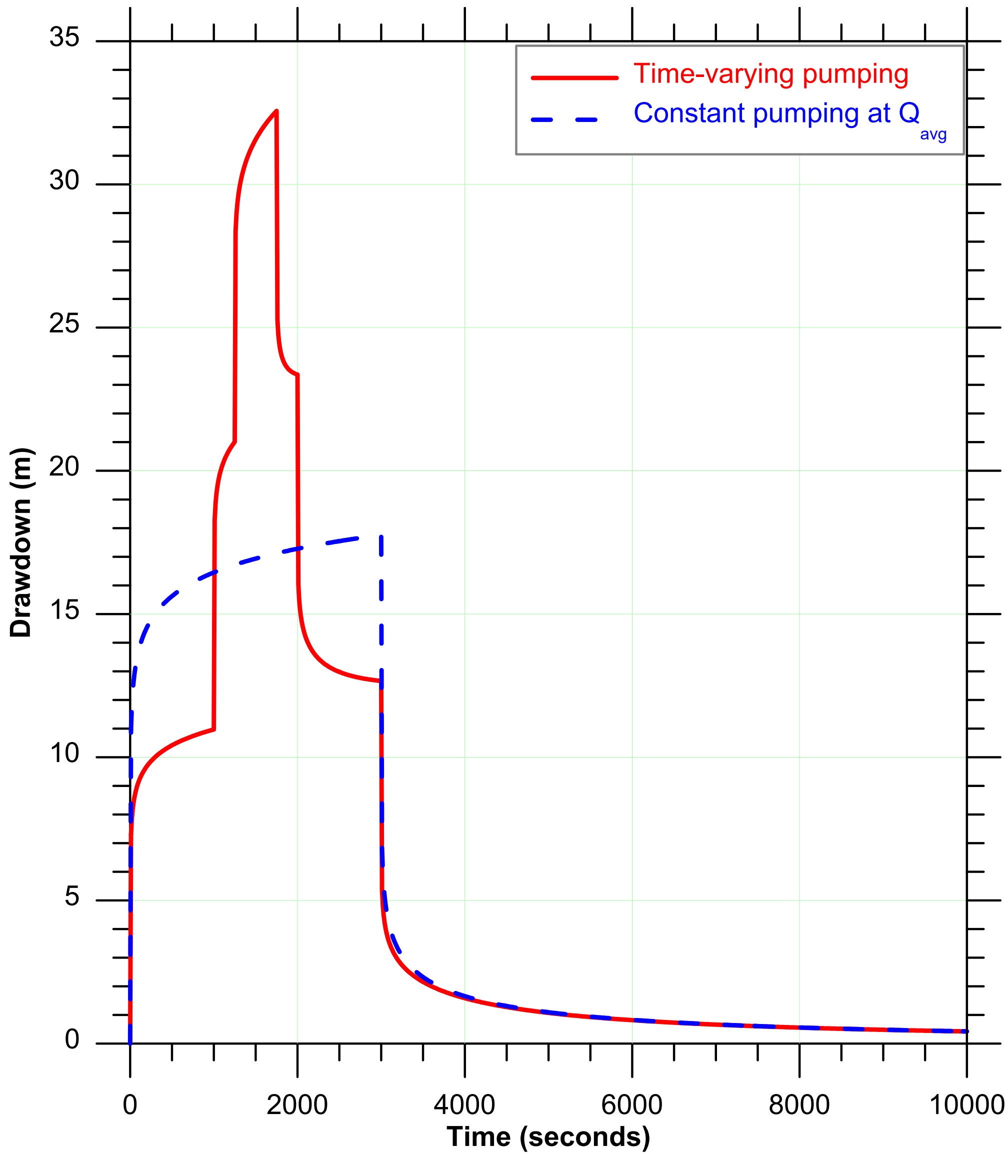


Figure 7. Drawdowns for the full duration of the pumping test

As shown in Figure 8, plotting the water levels during only the recovery period highlights the similarity of the water levels after the cessation of pumping. Although there are substantial variations in the pumping rate and calculated water levels during pumping, the responses during the recovery period are virtually indistinguishable.

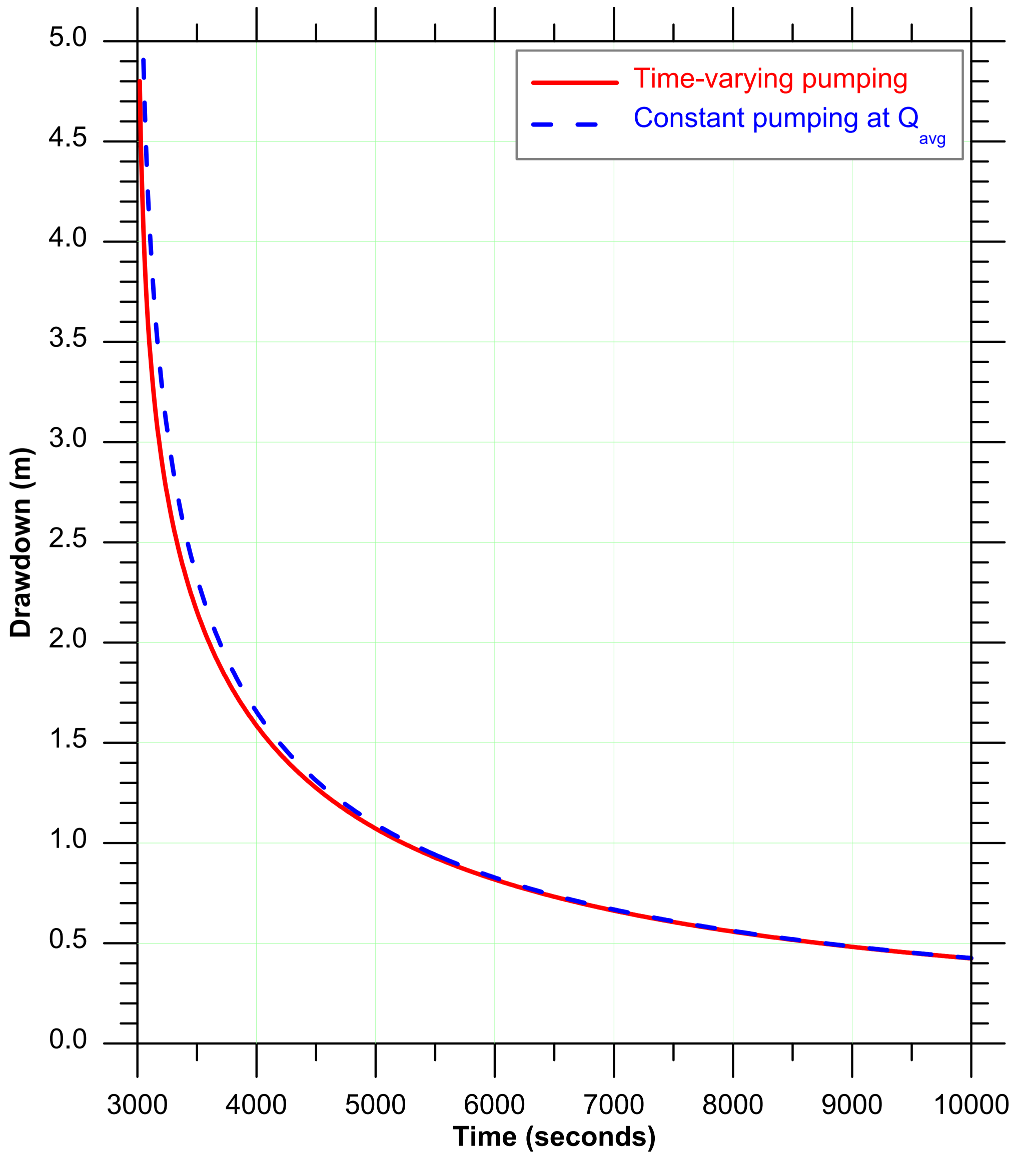


Figure 8. Comparison of recoveries for time-varying and constant pumping

3. Interpretation of recovery data: The principle of superposition

Almost all of the theoretical models that are applied to match the data from pumping tests are founded on the assumption that the aquifer response to pumping is linear. Mathematically, this means that neither the coefficients appearing in the governing equation nor the boundary conditions depend upon the drawdown. The property of linearity has important implications for the interpretation of pumping tests. For linear problems, solutions to complex problems can be derived by adding together known solutions to simpler ones. The adding of solutions is called *superposition*. Solutions are superimposed in time for interpreting recovery data.

Let us consider a well that is pumped at a constant rate Q for a duration t_{off}, followed by the monitoring of recovery. The drawdown and recovery data can be interpreted by breaking the problem into two parts, as shown in Figure 9.

- Part 1 - Drawdown period: pumping at a constant rate Q
- Part 2 - Recovery period: pumping continues at a constant rate Q, but another well starts pumping at t_{off} at a rate $-Q$.

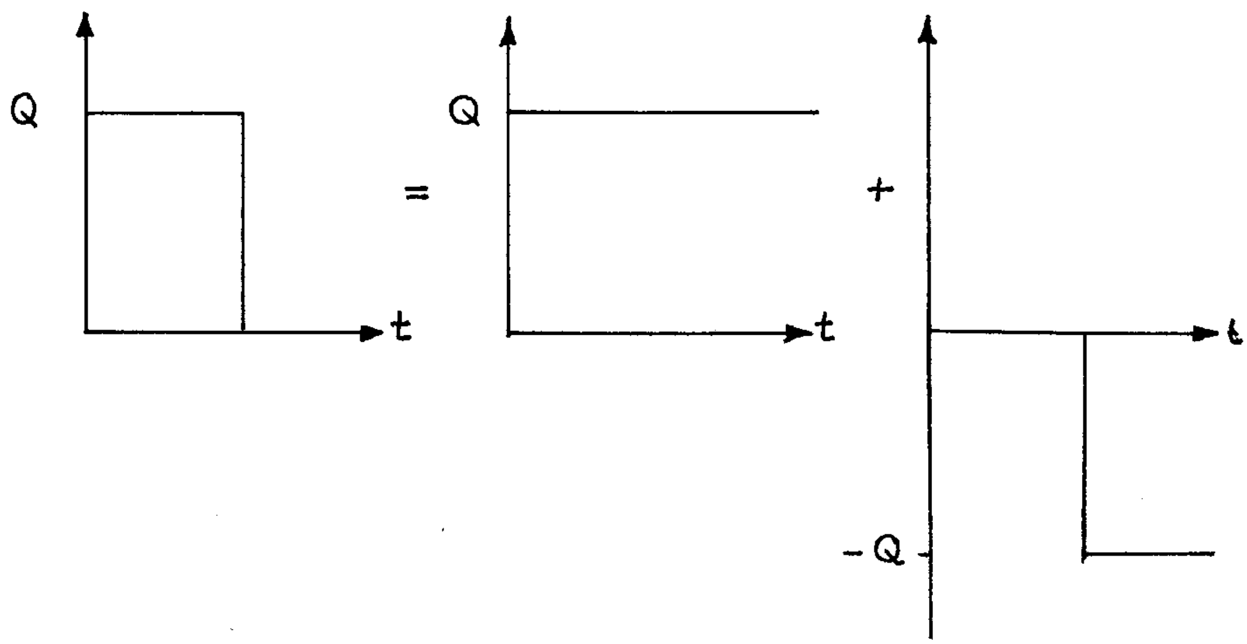


Figure 9. Illustration of superposition applied for a finite duration of pumping

The application of superposition for pumping from a fully penetrating well in an ideal confined aquifer is illustrated in Figure 10.

During pumping ($t \leq t_{off}$): $$s = \frac{Q}{4\pi T} W\left(\frac{r^2 S}{4Tt}\right)$$

During recovery ($t > t_{off}$): $$s = \frac{Q}{4\pi T} W\left(\frac{r^2 S}{4Tt}\right) - \frac{Q}{4\pi T} W\left(\frac{r^2 S}{4T(t-t_{off})}\right)$$

Here t is the total elapsed time since the start of the test and $(t\text{-}t_{off})$ is the elapsed time since the end of pumping, Q is the pumping rate, T is the transmissivity, S is the storage coefficient and r is the distance between the pumping well and the observation well.

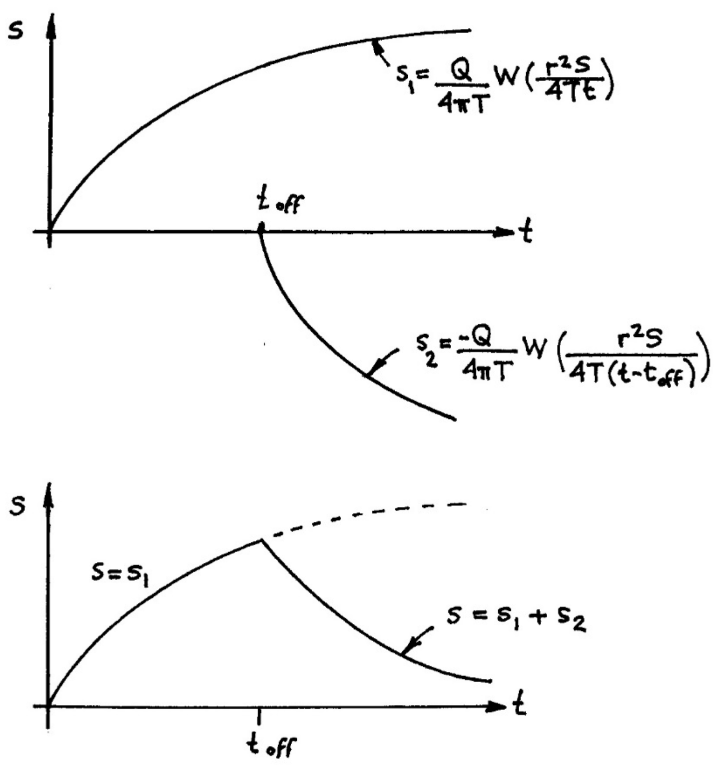


Figure 10. Application of the Theis (1935) solution for a finite duration of pumping

Although superposition is illustrated here with the Theis solution, it is important to note that it can be applied for any linear aquifer model. For example, superposition can be applied to interpret recovery data from pumping tests in unconfined aquifers with the analysis of Neuman (1974), and in leaky aquifers with the models of Hantush and Jacob (1955), Hantush (1960), and Neuman and Witherspoon (1969).

4. Cooper and Jacob (1946) straight-line analysis of recovery data

The Cooper-Jacob straight-line analysis has a particularly straightforward implementation for recovery analysis. The analysis provides both a simple means of estimating the transmissivity and of diagnosing the recovery response.

Extension of the Cooper-Jacob approximation for a finite duration of pumping

Recalling the Cooper-Jacob approximation of the Theis well function:

$$W(u) = -0.5772 - ln\{u\}$$

the extension of the solution for the recovery period following pumping at a constant rate for a duration t_{off} is given by:

$$s = \frac{Q}{4\pi T}\left[-0.5772 - ln\left\{\frac{r^2 S}{4Tt}\right\}\right] - \frac{Q}{4\pi T}\left[-0.5772 - ln\left\{\frac{r^2 S}{4T(t-t_{off})}\right\}\right]$$

Collecting terms:

$$s = \frac{Q}{4\pi T}\left[-ln\left\{\frac{r^2 S}{4Tt}\right\} + ln\left\{\frac{r^2 S}{4T(t-t_{off})}\right\}\right]$$

Using the properties of the log function, this can be written as:

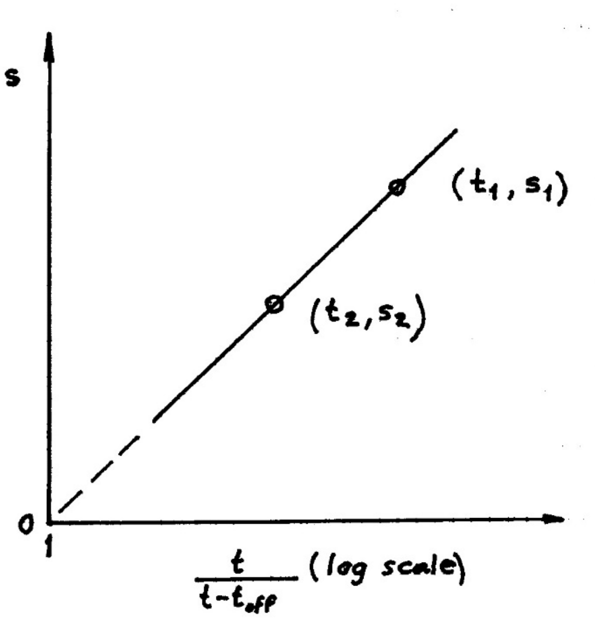
$$s = \frac{Q}{4\pi T} ln\left\{\frac{\left(\frac{r^2 S}{4T(t-t_{off})}\right)}{\left(\frac{r^2 S}{4Tt}\right)}\right\} = \frac{Q}{4\pi T} ln\left\{\frac{t}{t-t_{off}}\right\}$$

$$= 2.303 \frac{Q}{4\pi T} log_{10}\left\{\frac{t}{t-t_{off}}\right\}$$

This result implies that the late-time recovery depends only on the transmissivity, and not the storage coefficient.

Straight-line analysis

Let us consider two points along the plot of drawdown vs. $t/t\text{-}t_{off}$:



From the Cooper-Jacob solution we have:

$$s_1 - s_2 = \frac{Q}{4\pi T} 2.303 \left[\left[log_{10} \left\{ \left(\frac{t_1}{t_1 - t_{off}} \right) \right\} - \right] log_{10} \left\{ \left(\frac{t_2}{t_2 - t_{off}} \right) \right\} \right]$$

Solving for the transmissivity yields:

$$T = 2.303 \frac{Q}{4\pi} \frac{1}{(s_1 - s_2)} log_{10} \left\{ \frac{\left(\frac{t_1}{t_1 - t_{off}} \right)}{\left(\frac{t_2}{t_2 - t_{off}} \right)} \right\}$$

If $\left(\frac{t_1}{t_1 - t_{off}} \right)$ and $\left(\frac{t_2}{t_2 - t_{off}} \right)$ differ by a factor of ten (one log cycle), we denote s_1-s_2 as Δs and the Cooper-Jacob straight-line analysis reduces to:

$$\boxed{T = 2.303 \frac{Q}{4\pi} \frac{1}{\Delta s}}$$

Example 3

Let us consider an ideal confined aquifer that is pumped by a fully penetrating well. As shown in Figure 11, the results calculated with the Theis and Cooper-Jacob solutions are indistinguishable except at the beginning and end of pumping.

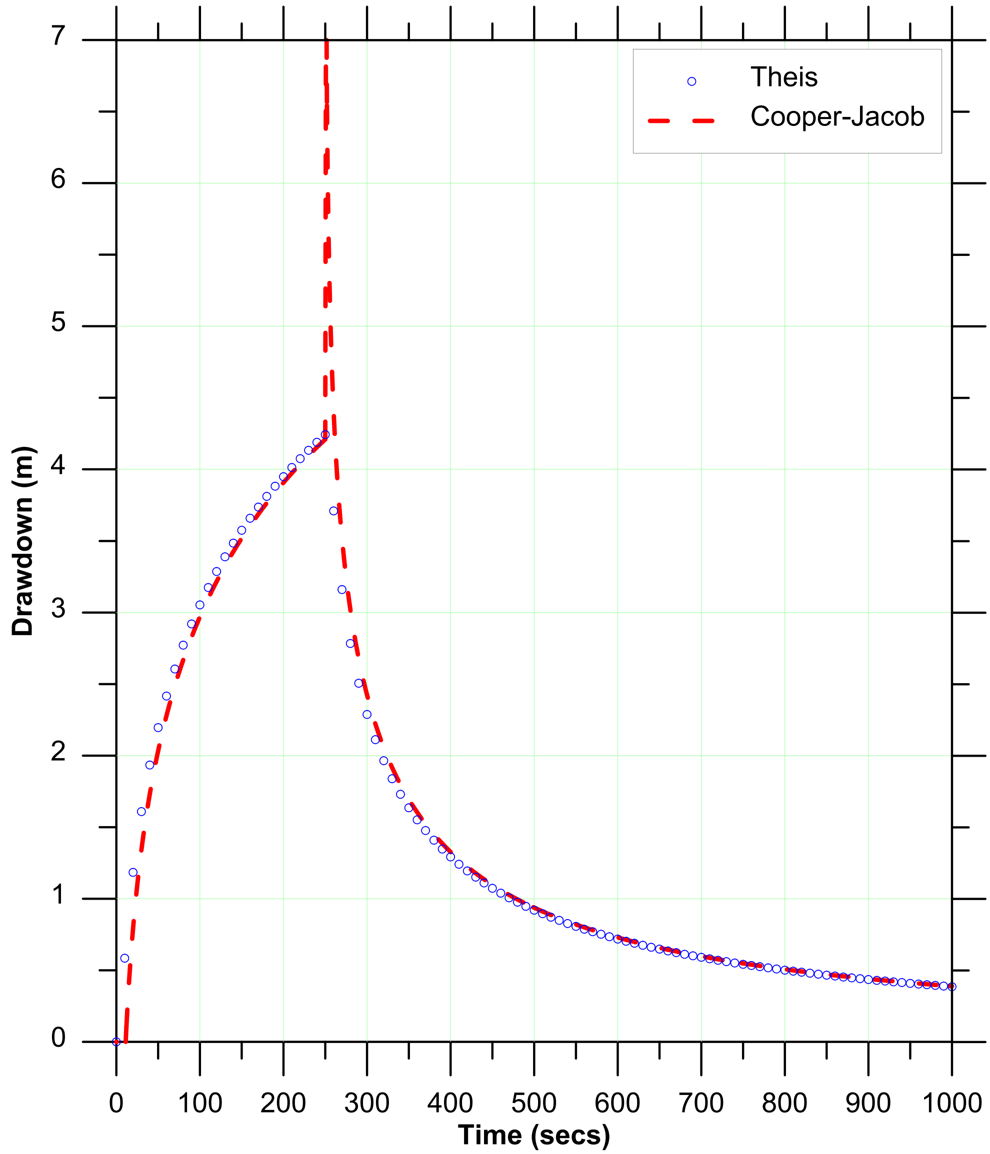


Figure 11. Theis solution and Cooper-Jacob approximation for Exmple 3

Let us re-plot the recovery portion of the data, with an alternate time axis, $\left\{\frac{t}{t-t_{off}}\right\}$. With this revised time axis, the end of pumping corresponds to a relatively large number and the recovery progress leftwards in Figure 12. As recovery continues, the drawdowns calculated with the Cooper-Jacob approximation converge with the Theis solution. It is important to note that the theoretical drawdowns extrapolate back to zero, in the limit as $t/(t-t_{off}) \rightarrow 1$. If they do not, there is probably a temporal trend in the water level data that should be extracted prior to further analysis. This may also be diagnostic of the influence of a hydrologic boundary.

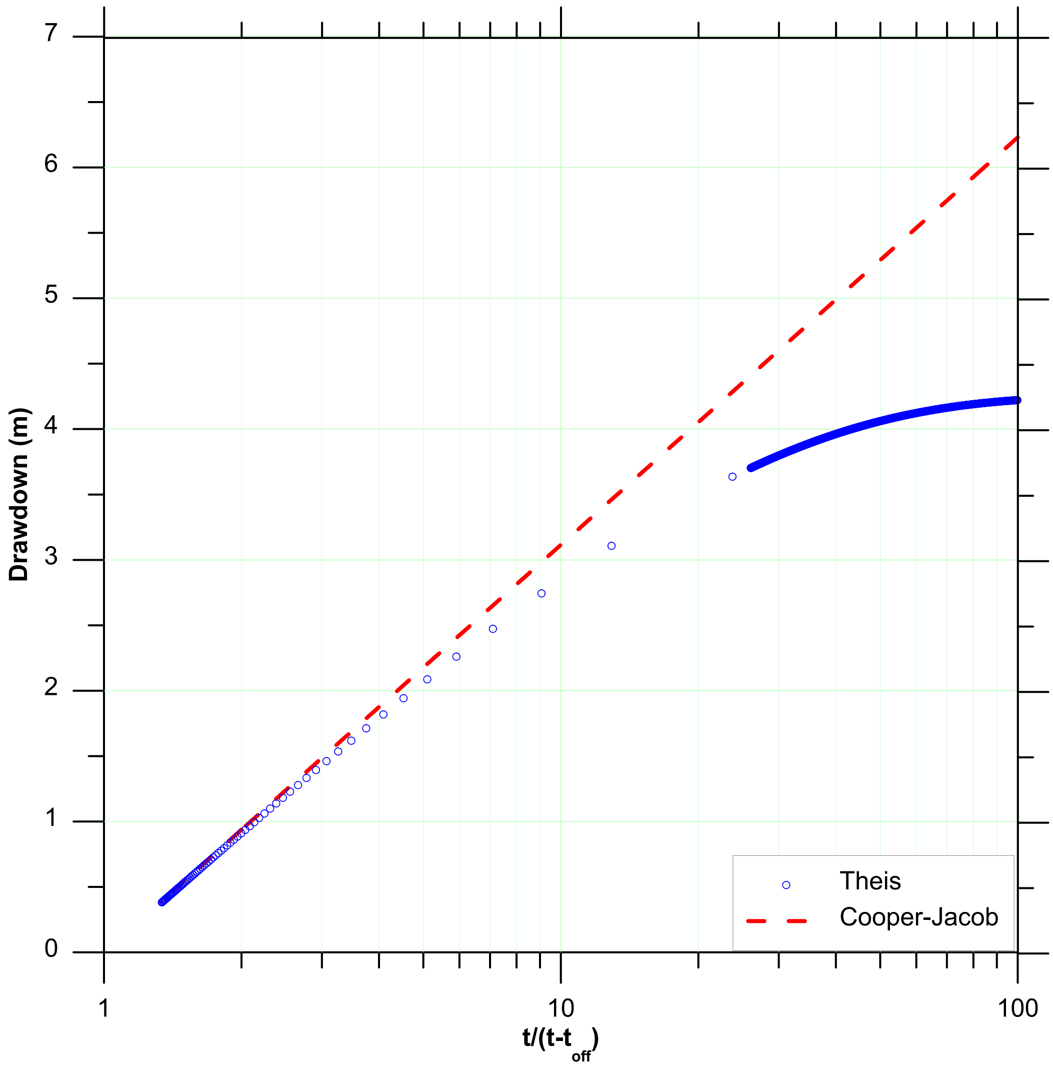


Figure 12. Theis solution and Cooper-Jacob solutions, semilog recovery plot

5. Diagnosis of aquifer structure from recovery data, Example 4

Recovery data are particularly useful for diagnosing boundary effects. These boundary effects may not be detectible in the drawdown portion of the test, but their manifestation in the recovery data may provide valuable insights into the structure of an aquifer.

We will consider the simple conceptual model shown in Figure 13 to illustrate the potential utility of recovery data for diagnosing complexities in aquifer structure. We consider a confined circular aquifer that is homogeneous and isotropic. We depart from the ideal Theis model in considering an aquifer that has a finite extent. Three cases will be considered:

1. Radially infinite aquifer (Theis);
2. Radially bounded aquifer, Constant-head at R = 1000 m (Theis-CH); and
3. Radially bounded aquifer, No-flow at R = 1000 m (Theis-NF).

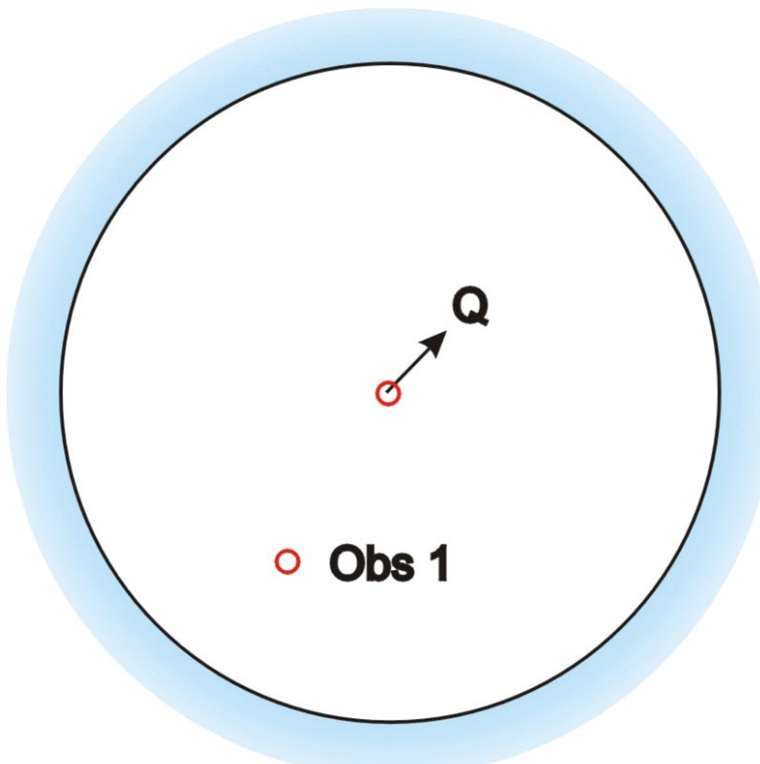


Figure 13. Conceptual model for an aquifer of finite radial extent

For the example calculations, a transmissivity and storativity of 6.944×10^{-2} m^2/min and 1.0×10^{-4} are assumed. The pumping rate is 1.0 m^3/min and the well is pumped for 120 minutes. Drawdowns are calculated at an observation well located at r = 10.0 m. As shown in Figure 14, the drawdowns calculated for all three cases are essentially the same. If we did not know that the aquifer was bounded, we would not be able to detect it from a pumping test of this duration.

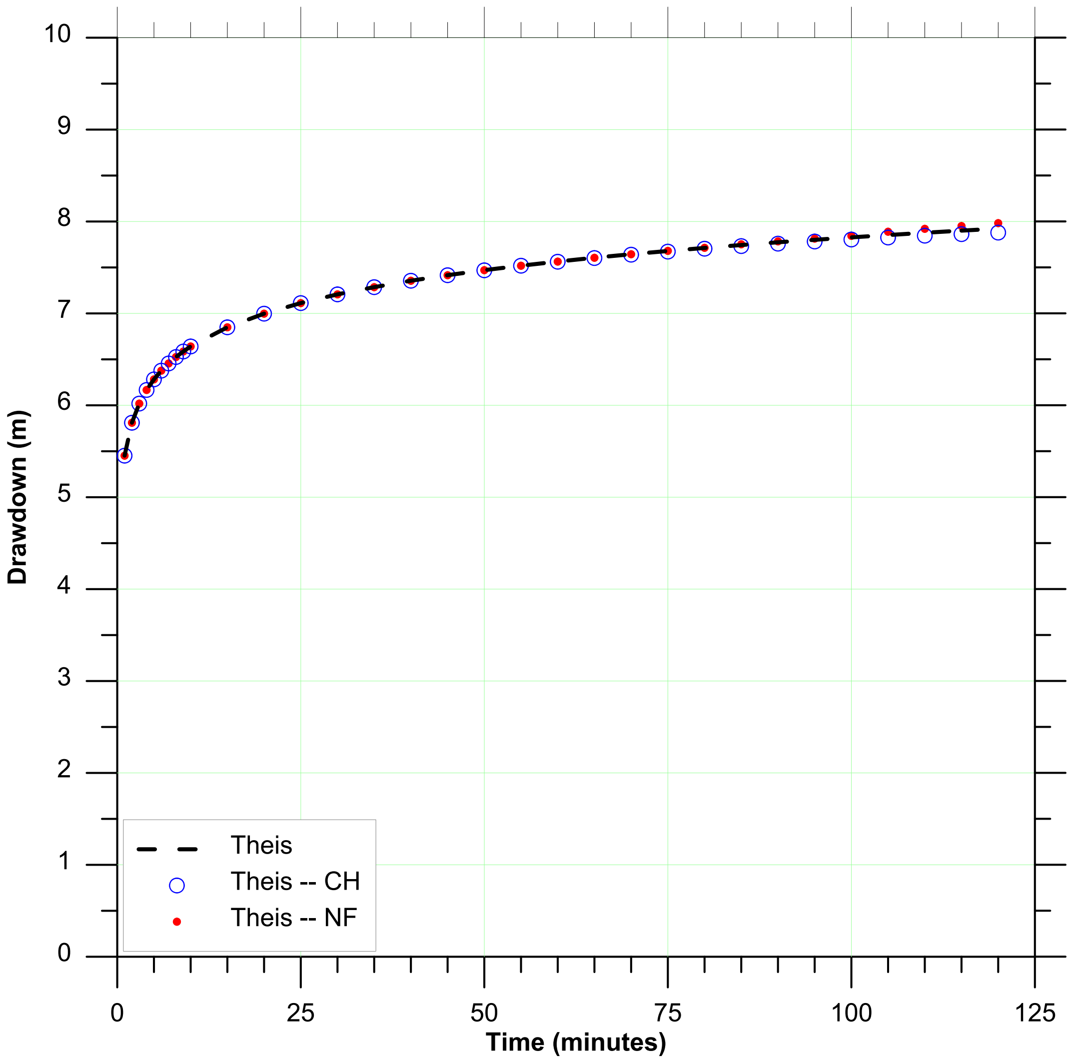


Figure 14. Calculated drawdowns during the pumping period

Calculated drawdowns versus time for the complete pumping and recovery period are plotted for the three cases in Figure 15. Although the drawdowns during the pumping period are nearly identical, there are systematic differences in the drawdowns during the recovery period. If we were fitting the complete drawdown record with the Theis solution for an infinite aquifer, we would notice that we could match the data from the pumping period but not the data from the recovery period. For a constant-head boundary (Theis-CH), the aquifer recovers too quickly. For a no-flow boundary (Theis-NF), the aquifer never recovers completely.

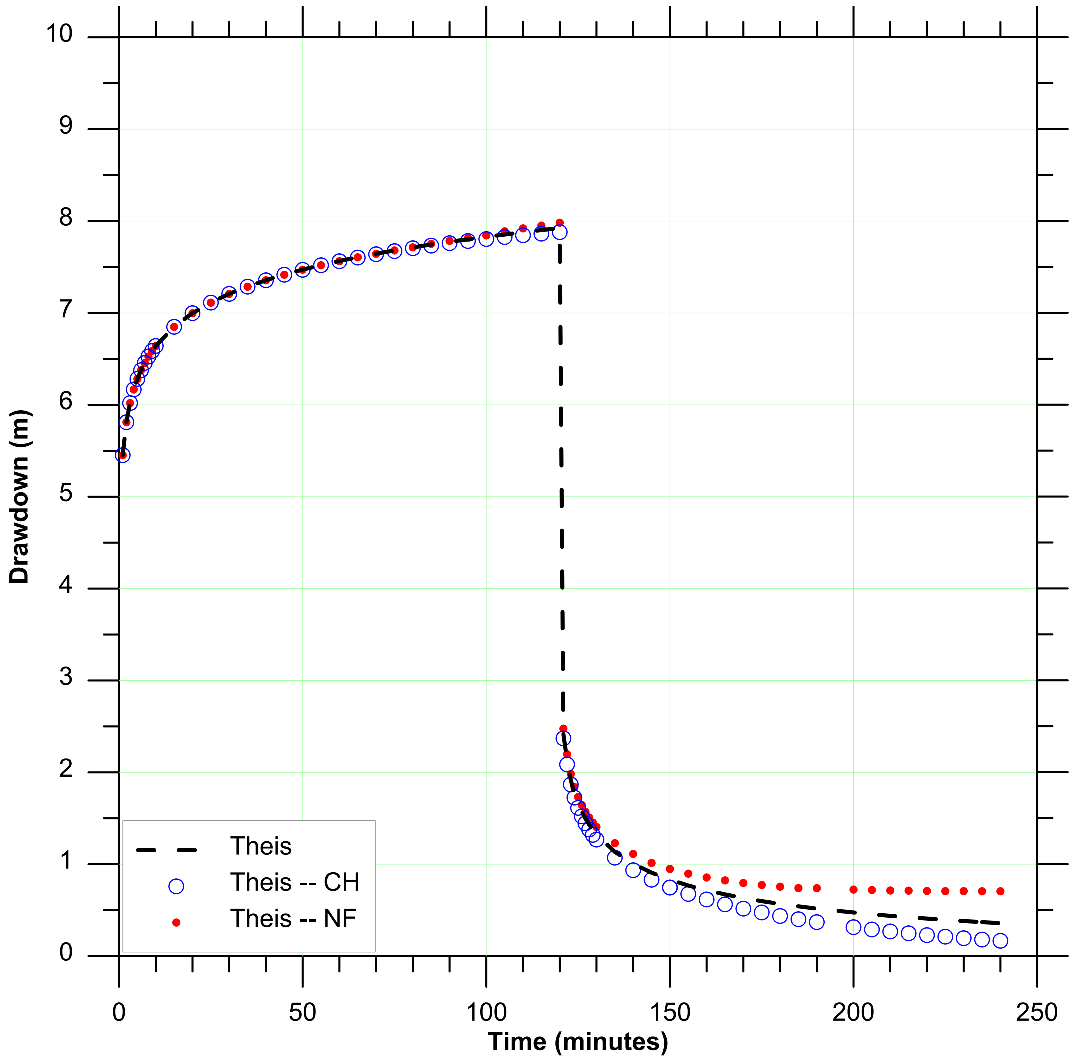


Figure 15. Calculated drawdowns during pumping and recovery

The recoveries calculated for the three cases are shown on Cooper-Jacob semilog plot in Figure 16. The results are highly diagnostic. The long-term recoveries for the bounded aquifers do not approach zero drawdown as $t/t\text{-}t_{\mathrm{off}}$ approaches 1.0. In the case of the aquifer surrounded by a surface of zero-drawdown (constant-head), the aquifer recovers completely before $t/(t\text{-}t_{off})$ approaches a value of 1.0. In the case of the aquifer surrounded by an impermeable boundary (no-flow), the aquifer never fully recovers.

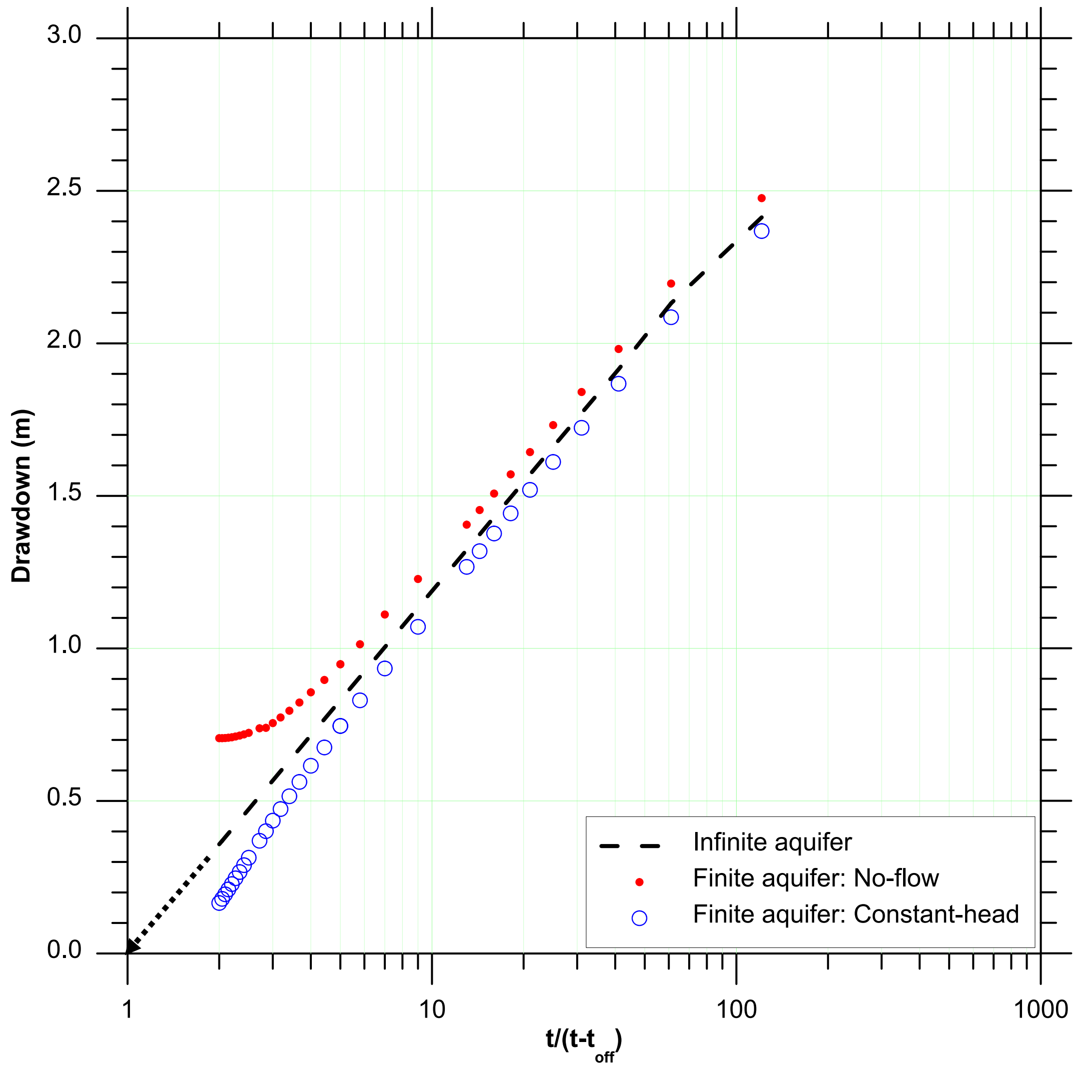


Figure 16. Semilog recovery plot

6. The van der Kamp (1989) method to extend the effective duration of pumping

van der Kamp (1989) introduced a different approach for working with recovery data. Although his approach is founded on classical superposition theory, it nevertheless represents a very useful extension of existing methods. van der Kamp's method appears to have been overlooked. As far as we are aware, it has not been implemented in any of the widely used interpretation packages. In our opinion this is an important oversight, and this section of the notes has been prepared in part to renew interest in this approach.

6.1 Theory of the van der Kamp method

For a general linear conceptual model, the drawdown *s(r,t)* caused by pumping at a variable rate *Q(t)* can be written as:

$$s(r,t) = \int_0^t Q(\tau)\ \ G(r, t-\tau)d\tau$$

This general relation is referred to as a convolution integral; the term *G(r,t)* represents the drawdown at a distance *r* caused by pumping for an instant at time *t* = 0, and is referred to frequently as the Green's function for a particular problem.

Let us consider an arbitrary pumping history represented by a set of discrete steps (Figure 17).

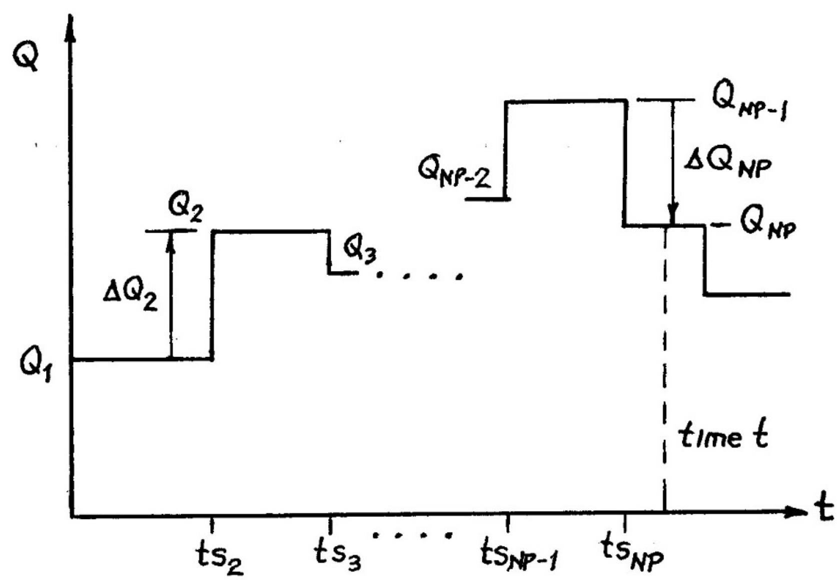


Figure 17. Discrete representation of an arbitrary pumping history

The pumping history for a discrete set of steps can be written as:

$$Q(t) = \sum_{i=1}^{NP(t)} \mathit{\Delta} Q_i H(t - ts_i)$$

Here *H* is the Heaviside step function, which is defined as:

$$\begin{aligned} H(t - ts_i) \; &= 0 \quad if \;\; t < ts_i \\ &= 1 \quad if \;\; t > ts_i \end{aligned}$$

The term ts_i designates the time at which the i^{th} change in the pumping rate occurs. At that time the pumping rate changes by an increment ΔQ_i. The term *NP(t)* designates the number of steps that have occurred up to any time *t*.

Substituting for the pumping history in the convolution integral yields:

$$s(r,t) = \int_0^t \left(\sum_{i=1}^{NP(\tau)} \Delta Q_i \, H(\tau - ts_i) \right) G(r, t - \tau) \, d\tau$$

Reversing the order of the summation and integration and using the properties of the Heaviside step function yields:

$$s(r,t) = \sum_{i=1}^{NP(t)} \Delta Q_i \int_0^{t - ts_i} G(r,t) \;\; d\tau$$

Expanding this relation yields:

$$\begin{aligned} s(r,t) = Q_1 \int_0^t G(r,\tau) d\tau + (Q_2 - Q_1) \int_0^{t - ts_2} G(r,\tau) d\tau + \cdot\cdot \\ \cdot + (Q_{NP} - Q_{NP-1}) \int_0^{t - ts_{NP}} G(r,\tau) d\tau \end{aligned}$$

Let us designate:

$$s_1(r,t) = Q_1 \int_0^t G(r,\tau) d\tau$$

The term s_1 represents the drawdown that would be observed at time *t* if the pumping rate had remained constant at a rate Q_1.

When we substitute for s_1, the expanded relation for the drawdown becomes:

$$s(r,t) = s_1(r,t) + (Q_2 - Q_1)\int_0^{t-ts_2} G(r,\tau)d\tau + \cdots$$
$$\cdot + (Q_{NP} - Q_{NP-1})\int_0^{t-ts_{NP}} G(r,\tau)d\tau$$

Making use of the definition of s_1, we can write this as:

$$s(r,t) = s_1(r,t) + \frac{(Q_2 - Q_1)}{Q_1} s_1(r, t - ts_2) + \cdots + \frac{(Q_{NP} - Q_{NP-1})}{Q_1} s_1(r, t - ts_{NP})$$

Rearranging to solve for s_1 yields:

$$s_1(r,t) = s(r,t) - \left[\frac{(Q_2 - Q_1)}{Q_1} s_1(r, t - ts_2) + \cdots + \frac{(Q_{NP} - Q_{NP-1})}{Q_1} s_1(r, t - ts_{NP})\right]$$

This is the general form of the van der Kamp (1989) algorithm for determining the equivalent drawdown that would be observed at any time if the pumping rate were constant at a rate Q_1.

6.2 Application of the van der Kamp method for pumping at a constant rate followed by recovery

Although the general form of the van der Kamp (1989) algorithm appears to be relatively complicated, it has a straightforward interpretation for the analysis of recovery following pumping a constant rate. For this case, during the recovery period we have $NP = 2$, $t_{s2} = t_{off}$, and $Q_2 = 0$, and van der Kamp's general form reduces to:

$$s_1(r,t) = s(r,t) + s_1\left(r, t - t_{off}\right)$$

This final result has a straightforward interpretation. If pumping had continued at a constant rate Q_1, the drawdown that would have been observed at any elapsed time, denoted s_1, would be equal to the actual drawdown plus the drawdown observed at time t-t_{off}.

Example

The van der Kamp approach is illustrated with an example calculation. We will consider a pumping well that penetrates the full thickness of an ideal confined aquifer. The assumed parameter values are listed below:

- Transmissivity, $T = 1.0\times10^{-4}$ m^2/sec;
- Storativity, $S = 1.0\times10^{-4}$;
- Pumping rate, $Q = 1.7\times10^{-3}$ m^3/sec;
- Duration of pumping, $t_{off} = 250$ seconds; and
- Radial distance, $r = 10$ m.

The calculated drawdown history is plotted in Figure 18.

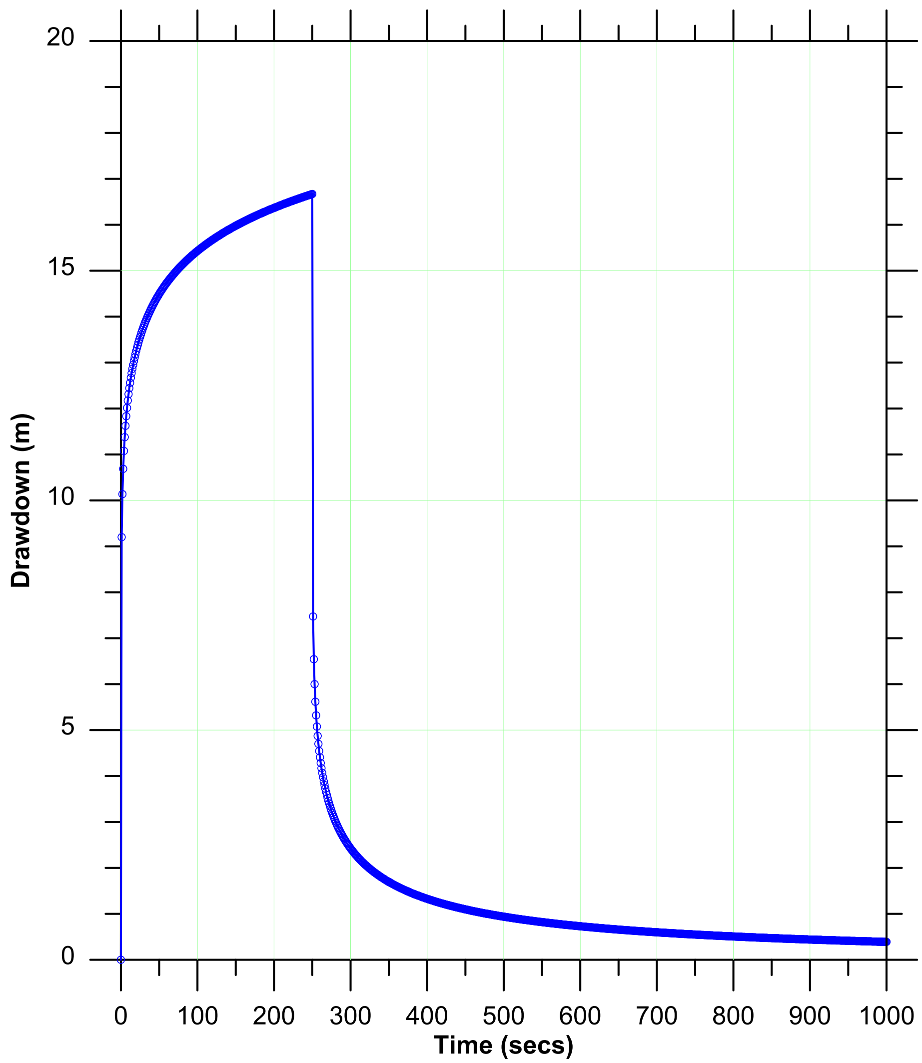


Figure 18. Calculated drawdowns during pumping and recovery

Let us estimate what the drawdowns would have been if the pumping had continued at a constant rate of 1.7×10^{-3} m^3/sec. To demonstrate the van der Kamp method, a hand calculation is made at t = 400 seconds.

Let us recall the general formula:

$$s_1(r,t) = s(r,t) + s_1(r, t - t_{off})$$

The pumping lasted 250 seconds; therefore, the equivalent constant-rate drawdown at t = 400 seconds is given by:

$$s_1(t = 400\ s) = s(t = 400\ s) + s_1(t - t_{off} = 400\ s - 250\ s = 150\ s)$$

From Figure 13, we observe from Figure 17 that the drawdown at 400 seconds is 1.33 m.

At t = 150 s, the well is still pumping; therefore $s_1 = s$ and we can again estimate the drawdown directly from Figure 17. The drawdown after 150 seconds is 15.98 m:

The equivalent drawdown is therefore given by:

$$s_1(t = 400\ s) = 1.33\ \mathrm{m} + 15.98\ \mathrm{m} = 17.31\ \mathrm{m}$$

This calculation is shown graphically in Figure 19. The red line is simply the pumping portion of the data, flipped in sign and shifted by the duration of pumping.

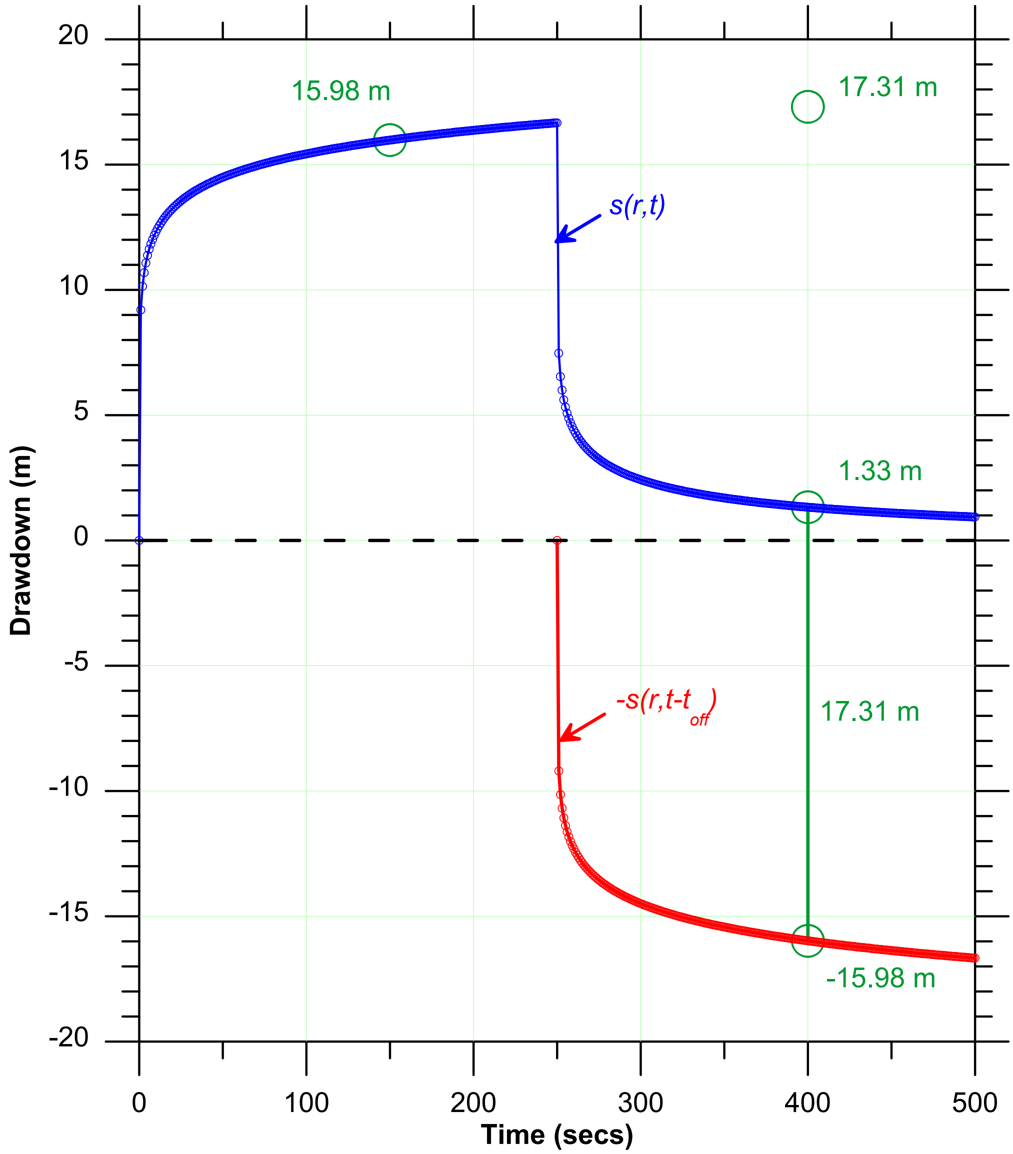


Figure 19. Calculation of equivalent constant-rate drawdown at 400 seconds

The results of applying the van der Kamp method for the entire drawdown record are shown in Figure 20. The drawdowns calculated with the Theis (1935) solution for continuous pumping at a constant rate of 1.7×10^{-3} m^3/sec are also plotted in Figure 20. The results obtained with the van der Kamp method match exactly the results obtained with the Theis solution.

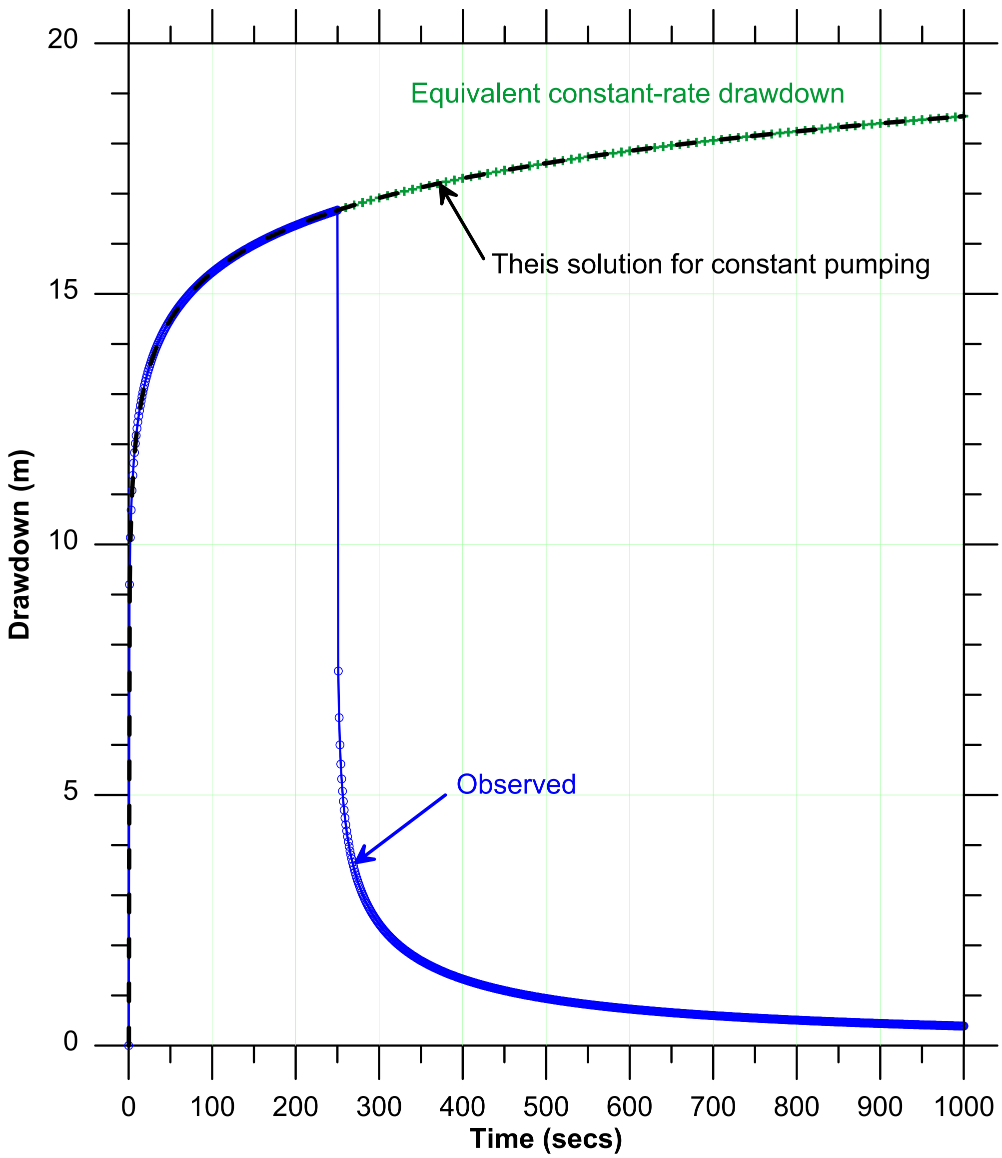


Figure 20. Actual drawdown and equivalent constant-rate drawdown

7. Case study: Estevan, Saskatchewan

In this case study, the van der Kamp (1989) method is applied to extend the effective duration of pumping of a pumping test conducted in a confined buried-channel aquifer near Estevan, Saskatchewan. The aquifer is described in Walton (1970), van der Kamp and Maathuis (2002), and Maathuis and van der Kamp (2003). Several pumping tests have been conducted in the aquifer; the data considered in this case study were obtained during a test conducted in 1984 and reported in van der Kamp (1985; 1989).

<u>Pumping test data</u>

- The aquifer was pumped at a constant rate for 41,520 minutes (about 29 days).

- Water levels following the end of pumping were monitored for an additional 249,000 minutes (173 days).

Drawdowns at observation well 11L-84 during the pumping and recovery periods are shown in Figure 21. For subsequent analysis, the original observations are supplemented with interpolated values indicated by the crosses. The interpolated values are taken from van der Kamp (1989; Table 2) and are smoothed slightly with respect to the original observations.

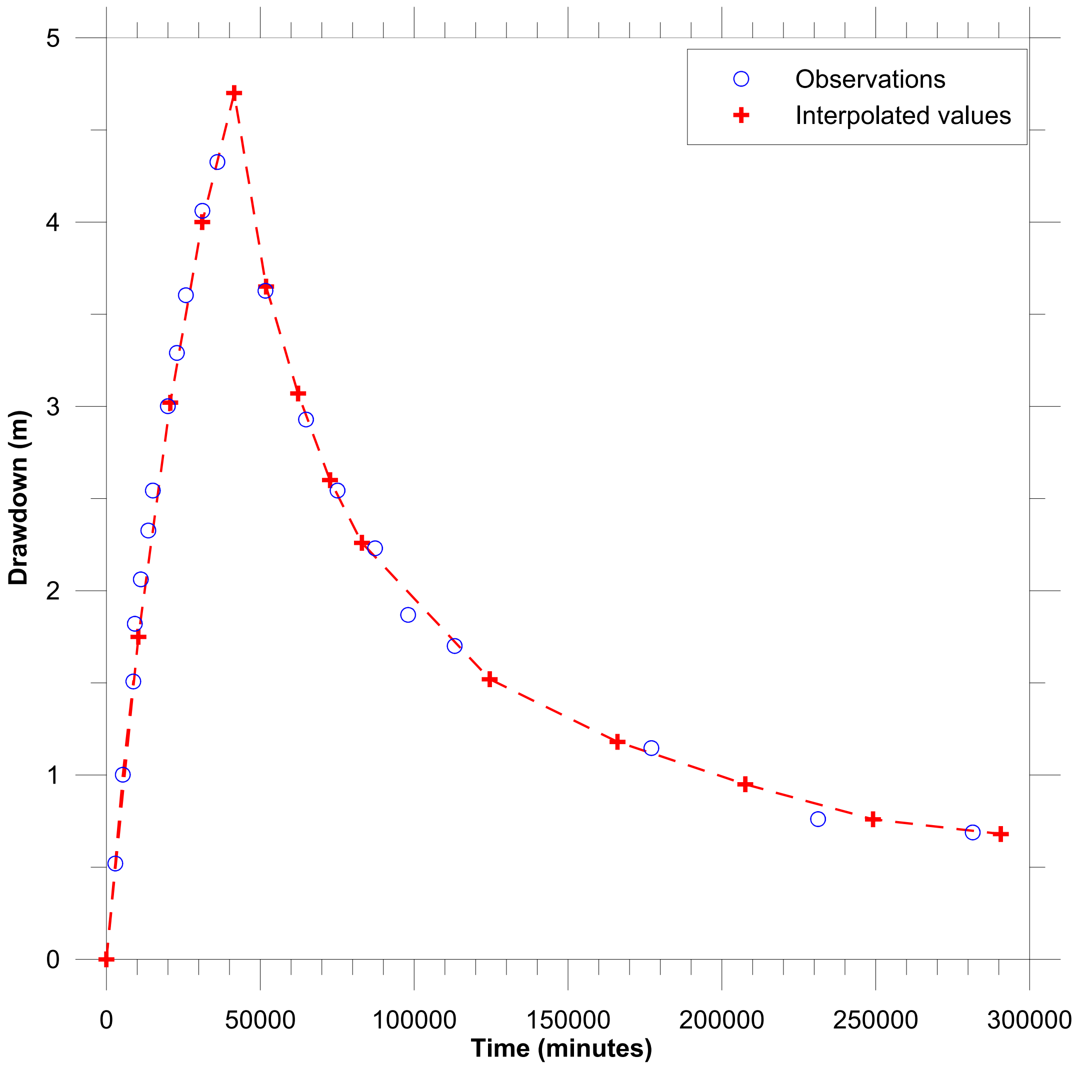


Figure 21. Raw drawdown data with interpolated values

Application of the van der Kamp (1989) method

For pumping at a constant rate followed by recovery, the drawdown that would be observed if the pumping had continued through the recovery period, $s_1(r,t)$, is calculated as:

$$s_1(r,t) = s(r,t) + s_1(r, t - t_{off})$$

Here $s(r,t)$ is the actual observed drawdown at elapsed time t, and t_{off} is the duration of pumping (t-t_{off} is the elapsed time since the end of pumping).

Extended drawdowns at three times beyond the end of pumping will be calculated. Recall that the well was pumped for 41,520 minutes.

t = 51,900 minutes

- Actual drawdown at t = 51,900 minutes: 3.65 m
- Elapsed time since the end of pumping: 51,900 – 41,520 = 10,380 minutes
- Constant-rate drawdown at t-t_{off} = 10,380 minutes: 1.75 m

The equivalent constant-rate drawdown is therefore:

$$s_1(r,t) = 3.65 + 1.75 = 5.40\ m$$

The results of the calculation are shown on Table 1. Using the recovery data from 10,380 minutes after the end of pumping, the effective duration of pumping is increased to 51,900 minutes.

Table 1. Extended results for *t* = 51,000 minutes

t (minutes)	s (t) (m)	t-t_{off} (minutes)	s (t-t_{off}) (m)	s_1 (m)
0	0.00			0.00
10380	1.75			1.75
20760	3.02			3.02
31140	4.00			4.00
41520	4.70			4.70
51900	3.65	10380	1.75	5.40

<u>t = 83,040 minutes</u>

- Actual drawdown at t = 83,040 minutes: 2.26 m
- Elapsed time since end of pumping: 83,040 – 41,520 = 41,520 minutes
- Constant-rate drawdown at $t\text{-}t_{off}$ = 41,520 minutes: 4.70 m

$$s_1(r,t) = 2.26 \text{ m} + 4.70 \text{ m} = 6.96 \text{ m}$$

Complete results up to 83,040 minutes are assembled on Table 2.

Table 2. Extended results to *t* = 83,040 minutes

t (minutes)	s (t) (m)	$t\text{-}t_{off}$ (minutes)	s ($t\text{-}t_{off}$) (m)	s_1 (m)
0	0.00			0.00
10380	1.75			1.75
20760	3.02			3.02
31140	4.00			4.00
41520	4.70	0		4.70
51900	3.65	10380	1.75	5.40
62280	3.07	20760	3.02	6.09
72660	2.60	31140	4.00	6.60
83040	2.26	41520	4.70	6.96

<u>t = 124,560 minutes</u>

- Actual drawdown at t = 124,560 minutes: 1.52 m
- Elapsed time since end of pumping: 124,560 – 41,520 = 83,040 minutes

A total elapsed time of 124,560 minutes corresponds to 83,040 minutes after pumping stopped, which is longer than the duration of pumping. Beyond an elapsed time of $2t_{off}$, we start to make use of the results for s_1 that have been assembled already. The calculations can be continued as long as there are measurable drawdowns.

- Constant-rate drawdown at t-t_{off} = 83,040 minutes: 6.96 m

$$s_1(r,t) = 1.52 \text{ m} + 6.96 \text{ m} = 8.48 \text{ m}$$

Complete results for all of the available drawdown data are assembled on Table 3 and are plotted in Figure 22.

Table 3. Results of equivalent constant-rate drawdown calculations

t (minutes)	s (t) (m)	t-t_{off} (minutes)	s (t-t_{off}) (m)	s_1 (m)
0	0.00			0.00
10380	1.75			1.75
20760	3.02			3.02
31140	4.00			4.00
41520	4.70	0		4.70
51900	3.65	10380	1.75	5.40
62280	3.07	20760	3.02	6.09
72660	2.60	31140	4.00	6.60
83040	2.26	41520	4.70	6.96
124560	1.52	83040	6.96	8.48
166080	1.18	124560	8.48	9.66
207600	0.95	166080	9.66	10.61
249120	0.76	207600	10.61	11.37
290640	0.68	249120	11.37	12.05

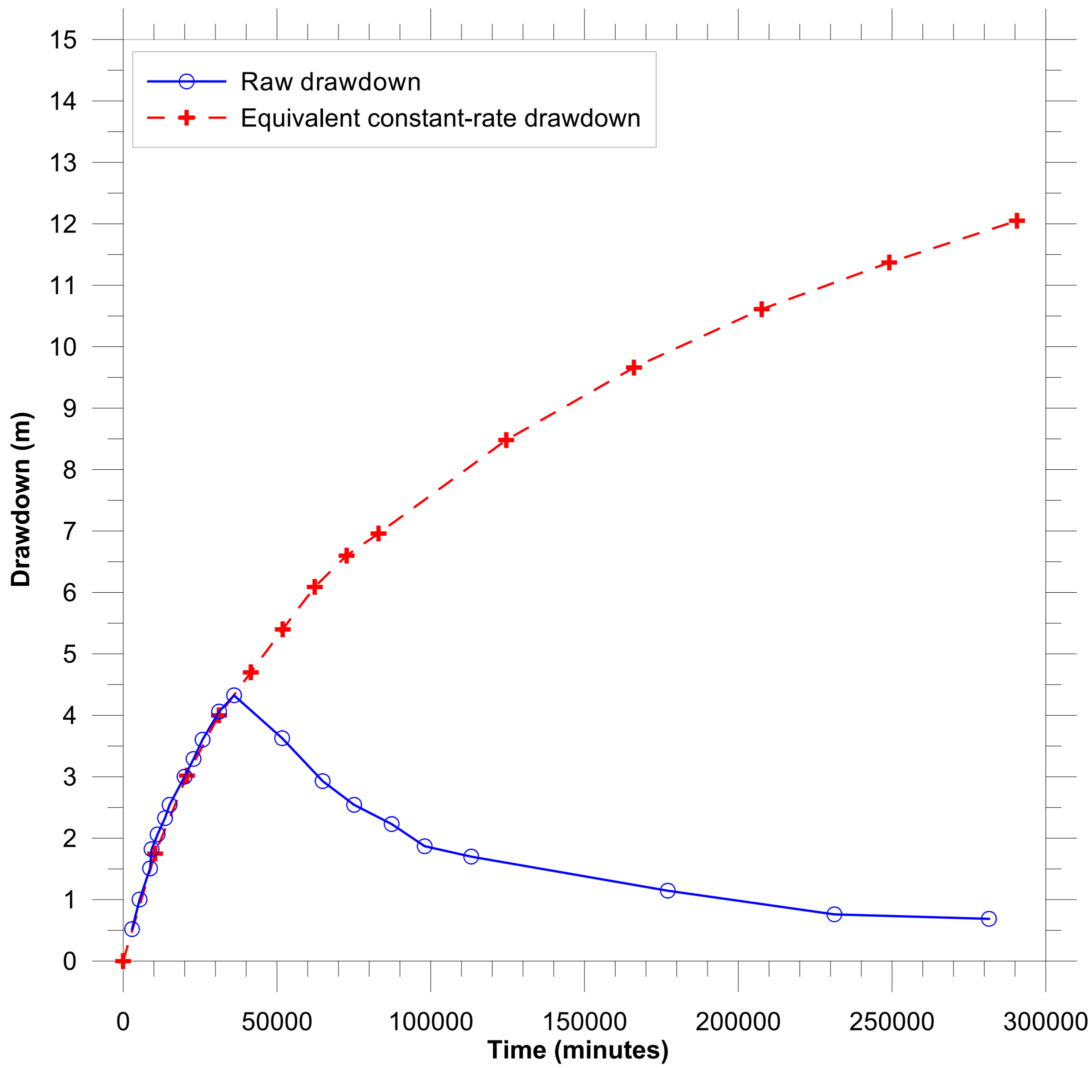


Figure 22. Complete record of equivalent constant-rate drawdowns

In the Estevan case study, the use of recovery data with the van der Kamp method extends the effective duration of pumping from one month to more than six months. The drawdown observed at the end of pumping is 4.70 m. The equivalent constant-rate drawdown for the last recorded water level is 12.05 m. The implications of this extension are best illustrated by plotting the raw drawdowns and the equivalent constant-rate drawdowns against the logarithm of time, as shown in Figure 23. We see that even after 29 days of pumping, it is possible to only identify the beginning of the long-term trend of the drawdown. In contrast, the accelerating trend that is characteristic of a buried-channel aquifer is clearly evident in the equivalent constant-rate drawdowns.

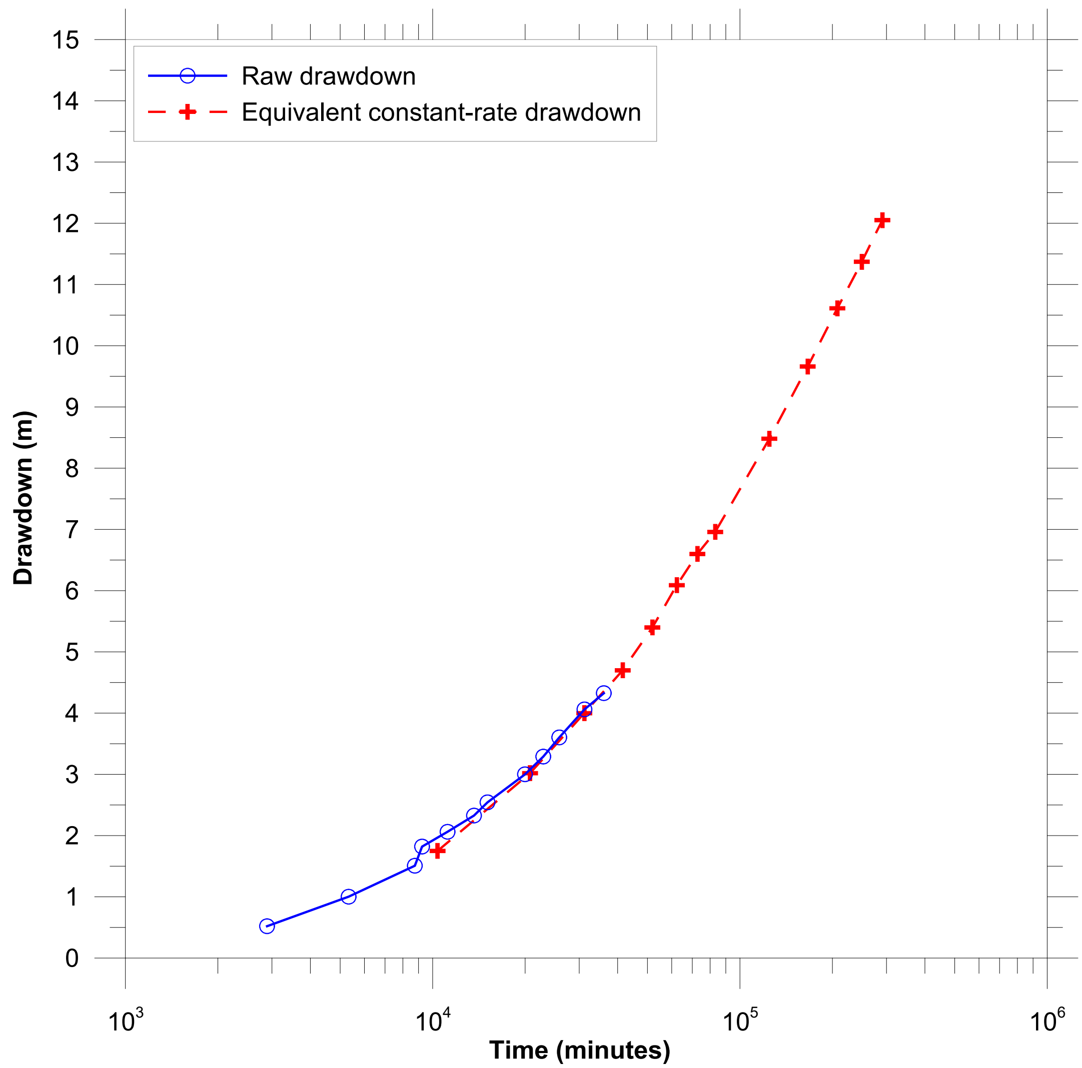
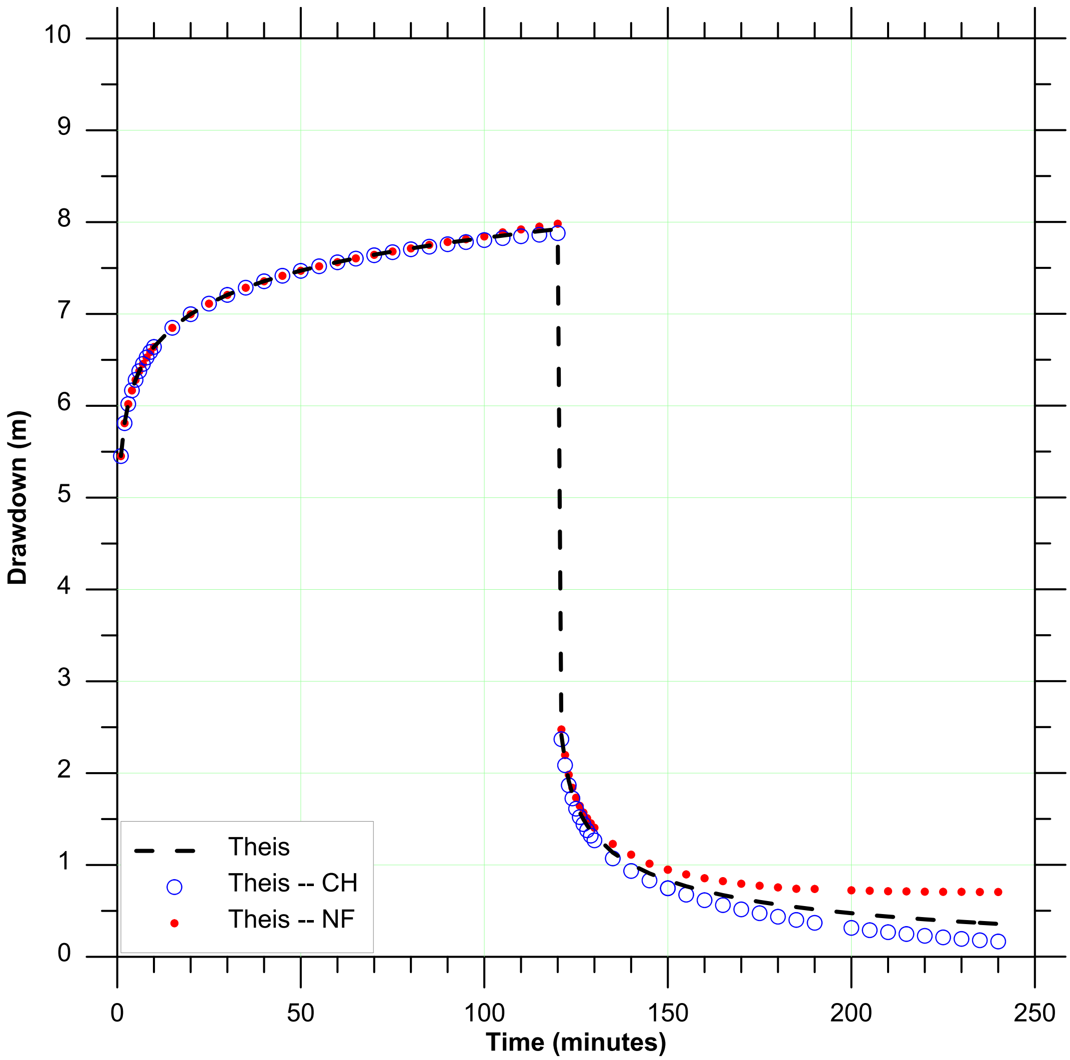


Figure 23. Semilog plot of equivalent constant-rate drawdowns

8. Application of the van der Kamp method for the bounded circular aquifer example

The utility of the van der Kamp method is further demonstrated by revisiting the example of the bounded circular aquifer. The complete drawdown histories from Figure 15 are reproduced below.



The equivalent drawdowns for extended pumping for the three cases are plotted in Figure 24. As discussed previously, it is not possible to detect the presence of the aquifer boundary with only the data from the pumping portion of the test, up to an elapsed time of 120 minutes. However, the boundary effects are clearly evident when the van der Kamp method is used to extend the effective duration of pumping. For the case of a zero-drawdown outer boundary (CH, constant-head), there is a distinct flattening of the drawdown beyond 120 minutes. In contrast, for the case of an impermeable boundary (NF, no-flow), the drawdowns accelerate beyond 120 minutes.

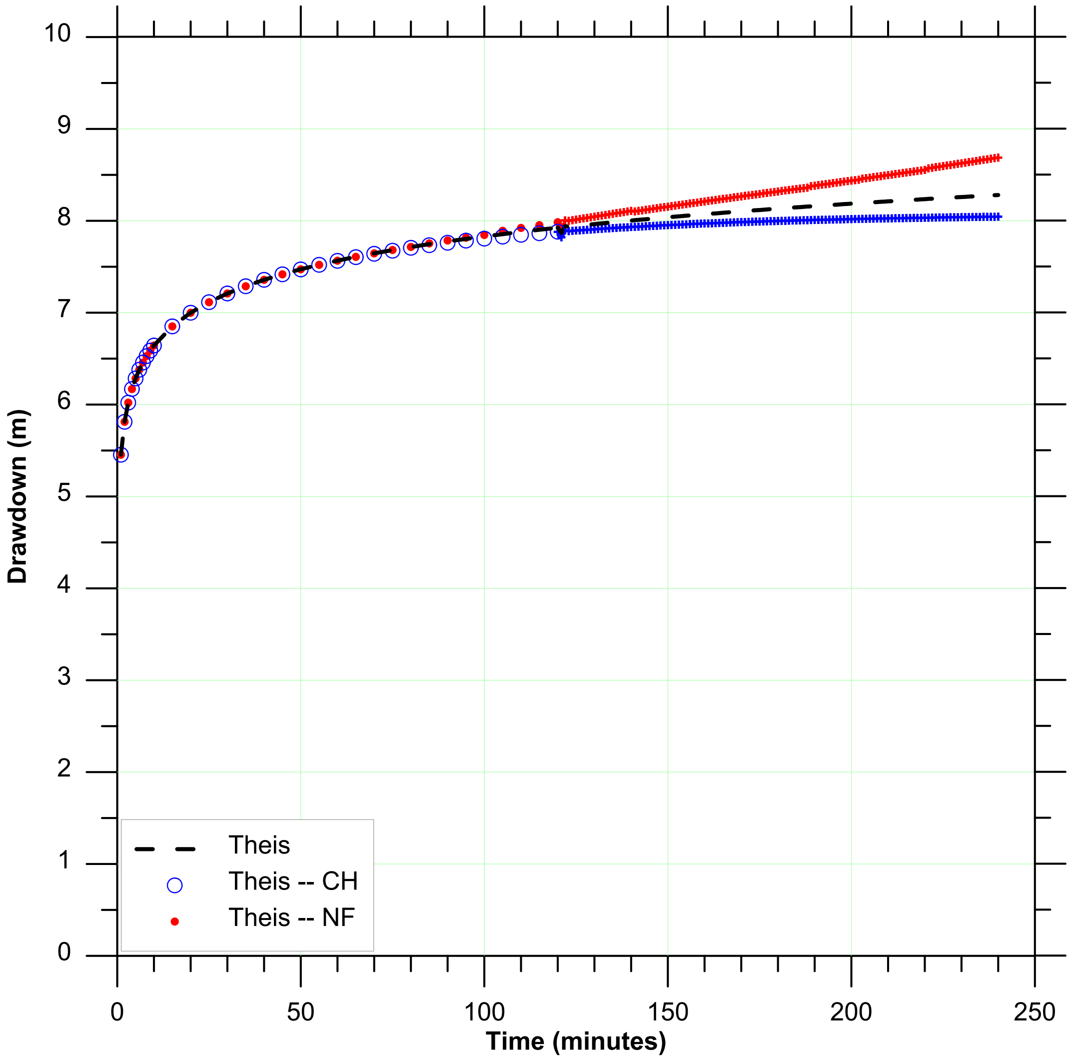


Figure 24. Equivalent constant-rate drawdowns

As shown in Figure 25, when the drawdowns for the extended duration of pumping are assembled on a semilog plot, the boundary effects manifest themselves as a deviation from the straight-line response.

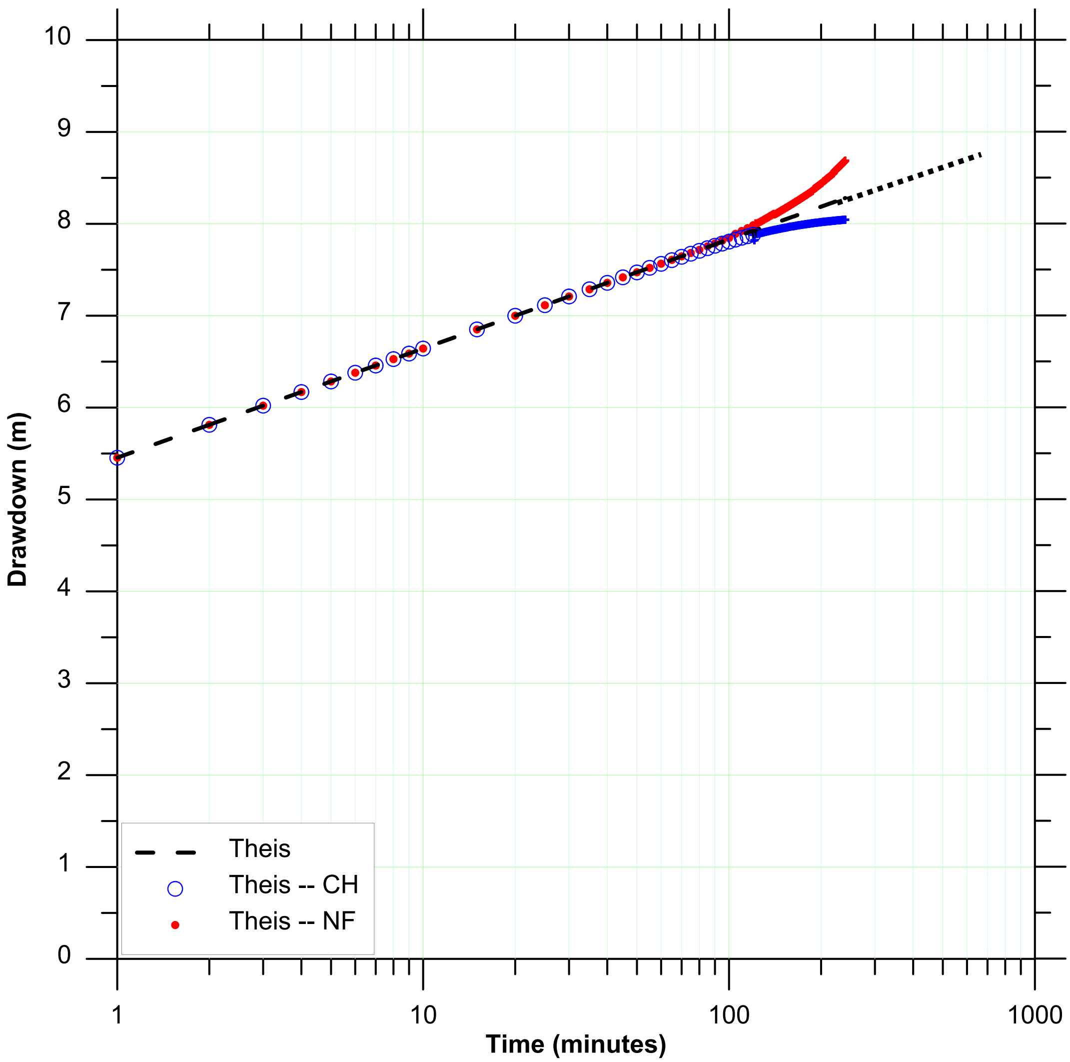


Figure 25. Equivalent constant-rate drawdowns, semilog plot

7. Summary of key points

1. Recovery data may be some of the best aquifer testing data available.

2. Recovery data are straightforward to interpret using superposition.

3. The Cooper-Jacob semilog plot of recovery data has useful diagnostic features.

4. The van der Kamp (1989) method of adjusting recovery data is a simple and illuminating technique for creating equivalent constant-rate pumping test data. The method can be used to effectively extend the duration of pumping.

5. Recovery data may offer insights into aquifer behavior not available from drawdown data.

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The significance and interpretation of recovery data: Additional readings

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Calculation of Constant-Rate Drawdowns from Stepped-Rate Pumping Tests

by G. van der Kamp[a]

ABSTRACT

Drawdown and recovery data obtained for stepped-rate pumping tests can be used to calculate the drawdowns that would occur if the test were carried out a constant rate without stopping. The recovery phase of constant-rate pumping tests can be analyzed by the same method because cessation of pumping can be treated as a step change of pumping rate. The calculation assumes only that pumping during each step is at a constant rate, and that the principle of superposition is applicable, i.e., that the ground-water system is linear and time-invariant. It does not depend on the availability of theoretical expressions for the drawdown due to pumping. The calculation can be carried out for as long as water-level measurements are continued; however, possible errors in the values of calculated drawdown increase with increasing time, thus limiting the practical length of time for which the calculated values are reliable.

The constant-rate drawdown curves characterize the response of the linear time-invariant ground-water system to pumping. They can be used for the determination of formation parameters if an appropriate theoretical model is available. They can also be used directly to predict drawdowns, and the scope of this application can be broadened by use of the reciprocity principle. In either case the use of recovery data can significantly extend the effective duration of pumping tests.

INTRODUCTION

In the theory and practice of aquifer pumping tests, many methods have been developed for analyzing drawdown data with changing pumping rates, and for analyzing recovery data for constant-rate tests (e.g., Theis, 1935; Moench, 1971; Reed, 1980). A common characteristic of most of these methods is that they depend on the prior assumption of a theoretical aquifer model (e.g., leaky artesian aquifer, Moench, 1971). The principle of superposition is then used to generate "customized" type curves for the particular pumping history of interest, and aquifer parameters are determined by matching the measured drawdown and recovery to these type curves.

An important drawback of this approach is that the assumed aquifer model may not be appropriate because of unrecognized hydrogeological factors such as aquifer boundaries. Because of the extra complications introduced by a changing pumping rate, deviation from the drawdown behavior predicted by the assumed aquifer model may not be readily apparent. This consideration, plus the effort involved in generating new type curves for each new problem, has probably contributed to the limited use of some of these methods. By contrast, a large body of analytical techniques and diagnostic experience is available for analyzing drawdown data from constant-rate pumping tests.

In this paper it is shown that recovery data for constant-rate tests, or drawdown and recovery data for stepped-rate tests can be used to calculate the hypothetical drawdown which would be observed were the test carried out at a constant rate without stopping. The proposed method assumes only that the principle of superposition is applicable; it does not require other assumptions

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concerning formation characteristics and geometry. The calculation is simple and can be readily carried out by means of a calculator or small computer.

The main advantage of the proposed method is that it allows analysis of drawdown and recovery data from stepped-rate and constant-rate pumping tests by means of the available analysis techniques for drawdown data from constant-rate tests and without the need to generate special sets of type curves. The use of recovery data as an integral part of the analysis means that the duration of the test is effectively extended beyond the duration of pumping.

Constant-rate drawdown curves characterize the response of a linear ground-water system to pumping or injection. For some cases there may be little point in calculating formation parameters by "force-fitting" the drawdown data to some theoretical model of the aquifer. In principle, the drawdown curves can be used directly to predict drawdown due to pumping. The linear theory of hydrologic systems (Dooge, 1973) can be utilized for this purpose, and its application can be broadened by use of the reciprocity principle (McKinley *et al.*, 1968).

The purpose of this paper is to show how constant-rate drawdown curves can be calculated. The theory is presented and the method of calculation is illustrated by examples of actual pumping tests. The application of the drawdown curves in terms of linear system theory and the reciprocity principle is also briefly discussed. Possible limitations on use of the method are reviewed, but its practical usefulness can only be established by extensive field testing.

THEORY

Calculation of Constant-Rate Drawdown Curves

The principle of superposition can be applied to any ground-water system which is linear and time-invariant (Bear, 1979, p. 152; Dooge, 1973, p. 85). Linearity is implicit in the basic ground-water flow equations, i.e., Darcy's Law and constant proportionality between change of head and release of water from storage. The condition of time-invariance means that permeability and specific storage must be constant, and that there must be no change with time in the geometry of the system (e.g., no dewatering). These conditions are assumed for nearly all available pumping-test theory and presumably are widely applicable.

Suppose that a production well is pumped without stopping at constant rate Q_0, starting at time $t = 0$. The resulting constant-rate drawdown curve for a given observation point will be denoted by $s_0(t)$. Next, suppose instead that at time $t = t_1$, the pumping rate is changed to Q_1. This change can be treated as if the well continued to be pumped at rate Q_0 and at t_1 additional pumping at a rate $Q_1 - Q_0$ is superimposed (see Figure 1). Similarly, if at $t = t_2$ the rate is changed to Q_2 the change can be treated as additional pumping at a rate $Q_2 - Q_1$. Note that $Q_1 - Q_0$ and $Q_2 - Q_1$, etc. can be negative. By application of the superposition principle, the total drawdown due to the stepped-rate pumping is

$$s(t) = s_0(t) + \frac{Q_1 - Q_0}{Q_0} s_0(t - t_1) + \frac{Q_2 - Q_1}{Q_0} s_0(t - t_2) + \ldots \quad \ldots\ldots(1)$$

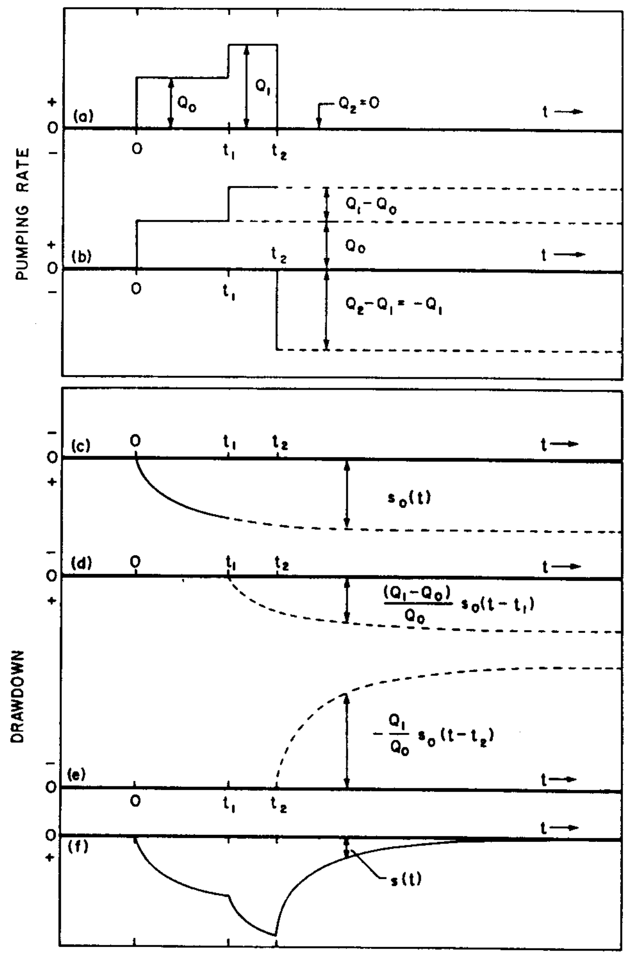

Fig. 1. Illustration of superposition as expressed by equation (1) for a two-step pumping test with subsequent recovery: (a) actual pumping rate; (b) representation of actual pumping rate as sum of three continuing components; (c) drawdown due to pumping at rate Q_0; (d) drawdown due to pumping at rate $Q_1 - Q_0$; (e) drawdown (recovery) due to pumping (injection) at rate $-Q_1$; (f) resultant drawdown due to actual pumping rate (sum of c, d, and e).

where $s(t)$ = total drawdown; $s_0(t)$ = constant-rate drawdown due to pumping at a constant rate Q_0 starting at $t = 0$; t = time since start of pumping; and Q_1, Q_2, etc are the constant pumping rates during the time intervals t_1 to t_2, t_2 to t_3, etc. The physical meaning of equation (1) is illustrated in Figure 1. Note that $s_0(t) = 0$ for t less than or equal to zero.

The purpose of this paper can now be stated more precisely: it is to show how the constant-rate drawdowns $s_0(t)$ can be calculated from the drawdowns $s(t)$ measured during the pumping and recovery phases of stepped-rate pumping tests.

It is important to note that any "constant-rate" pumping test of finite duration can be considered as a stepped-rate test because the end of pumping is essentially a step change of pumping rate. This particular application of the theory is discussed in more detail in the following section.

Using summation notation, equation (1) leads to:

$$s_0(t) = s(t) - \sum_{j=1}^{n} \frac{(Q_j - Q_{j-1})}{Q_0} s_0(t - t_j) \qquad (2)$$

where n = the number of pumping-rate steps; and $s(t)$ is the drawdown, i.e., the measured deviation of water level from the static water level. If water-level measurements are taken during the recovery phase (i.e., after pumping ceases), then $Q_n = 0$.

The actual calculation of $s_0(t)$, using equation (2), proceeds for successive time intervals, each of duration t_1, as follows:

1. $0 \leqslant t < t_1$. For this time interval $s_0(t - t_1)$, $s_0(t - t_2)$, etc. are equal to zero, thus:

$$s_0(t) = s(t) \qquad (3)$$

2. $t_1 \leqslant t < 2t_1$. For this time interval the values of $s_0(t - t_1)$ have already been calculated in the previous interval because $t - t_1$ is less than t_1. Similarly the values of $s_0(t - t_2)$, $s_0(t - t_3)$, etc. will either be zero if $t_2 > 2t_1$, $t_3 > 2t_1$, etc. or these values will have been obtained from the first time interval. Thus $s_0(t)$ in this time interval can be calculated with equation (2), making use of the values of $s_0(t)$ computed for the previous time interval.

3. $2t_1 \leqslant t < 3t_1$, etc. For these time intervals $s_0(t)$ can again be calculated with equation (2) using the values of $s_0(t)$ calculated for the previous time intervals.

Equation (2) can be conveniently applied by using a constant time step Δt starting at time $t = 0$, and calculating $s_0(t)$ at successive multiples of Δt, so that the end results are values for $s_0(0)$, $s_0(\Delta t)$, $s_0(2\Delta t)$, $s_0(3\Delta t)$, etc. The value of Δt may be smaller than or equal to t_1. Interpolation must be used if the times of drawdown measurements or the times at which pumping rate changes occurred are not exact multiples of Δt. The final result of the calculation will be values of $s_0(t)$, the constant-rate drawdown, extending for as long as the measured water levels differ significantly from the static water level. The method of calculation is illustrated in the EXAMPLES section of this paper, using field data for a three-step pumping test.

The Special Case of Recovery After a Constant-Rate Test

The most common application of the proposed method is likely to be in the analysis of residual drawdown data obtained during the recovery phase of a constant-rate pumping test. As already pointed out, reducing the pumping rate to zero can be treated as a step change of the pumping rate. For this simple case ($Q_1 = 0$), equation (2) gives:

$$s_0(t) = s(t) + s_0(t - t_1) \qquad (4)$$

where t_1 = duration of pumping, and $s(t)$ = measured drawdown during and after pumping.

Suppose that significant residual drawdowns persist to a time equal to or more than p times, but less than p + 1 times, the duration of pumping:

$$pt_1 \leqslant t < (p + 1)t_1 \qquad p = 0, 1, 2, 3, \ldots . \qquad (5)$$

The term $s_0(t - t_1)$ in equation (4) can be related to the previous time interval by substituting $t - t_1$ for t in equation (4):

$$s_0(t - t_1) = s(t - t_1) + s_0(t - 2t_1) \qquad (6)$$

Combining equation (6) with equation (4) gives

$$s_0(t) = s(t) + s(t - t_1) + s_0(t - 2t_1) \qquad (7)$$

This iteration can be continued to $t - pt_1$, the time interval when the well was pumped. For this interval the measured drawdown is identical with the constant-rate drawdown, i.e., $s(t - pt_1) = s_0(t - pt_1)$. The iteration, therefore, leads to:

$$s_0(t) = s(t) + s(t - t_1) + s(t - t_2) + \ldots + s(t - pt_1) \qquad (8)$$

In summation notation, equation (8) can be written as:

$$s_0(t) = \sum_{k=0}^{p} s(t - kt_1) \qquad (9)$$

Equation (9) states that the drawdown which would have occurred if pumping had continued is just the sum of the actual measured drawdowns at intervals t_1 apart going back to the time interval during which the well was actually pumped. The calculation of the constant-rate drawdowns for the recovery phase of a constant-rate pumping test thus consists of a simple addition.

Equation (9) allows calculation of $s_0(t)$ for as long as water-level measurements are continued. However, the measured drawdown, s(t), is subject to the uncertainty of measurement errors and of natural changes of the static water level. If the possible error in the measured drawdown is e(t), then the actual value of the drawdown lies in the range $s(t) \pm e(t)$. If this possible error is included in equation (9), the result is:

$$s_0(t) = \sum_{k=0}^{I} s(t - kt_1) \pm E(t) \qquad (10)$$

where $$E(t) = \sum_{k=0}^{p} e(t - kt_1) \qquad (11)$$

E(t) is the total possible error in the value of $s_0(t)$.

Equations (10) and (11) show that the possible errors in measured drawdown accumulate in the calculation of the constant-rate drawdown $s_0(t)$. The possible error in the measured drawdown, s(t), will tend to increase with time due to increasing uncertainty of the natural static water level, and the value of s(t) will decrease with time because the water level is recovering. Thus, at some point in time, the possible error will become large compared to the measured drawdown and calculation of the constant-rate drawdowns $s_0(t)$ beyond this time will have little meaning. The use of recovery data for extrapolating drawdown curves should therefore always be accompanied by a realistic estimate of possible errors. This is especially important if hydrogeologic interpretations are to be made on the basis of minor inflections in the shape of the constant-rate drawdown curves.

An example of the use of recovery data for a constant-rate pumping test is given in the EXAMPLES section of this paper.

Use of Constant-Rate Drawdown Curves to Predict System Behavior

From the point of view of systems theory, the purpose of many pumping tests (or aquifer performance tests) is to obtain constant-rate drawdown curves for one or more observation points. In this paper it is shown how such curves can be obtained from stepped-rate tests and how they can be extended in time by use of data obtained during the recovery phase. The constant-rate drawdown curves obtained in this manner characterize the response of the ground-water system to pumping and can be used to analyze and predict the behavior of the system.

Probably the most well-known use of constant-rate drawdown curves is to determine aquifer parameters such as transmissivity by matching the measured curves to theoretical type curves (e.g., Theis, 1935; Kruseman and de Ridder, 1970; Reed, 1980). However, such applications are only meaningful if the theoretical model for which the type curves were calculated is a reasonable approximation of the actual hydrogeological conditions at the test site.

For some cases the constant-rate drawdown curves themselves can be used directly to describe and predict the response of a ground-water system to pumping. The hydrogeological setting of many ground-water systems is too complex to be described by a few formation parameters, and the empirical constant-rate drawdown curves for such systems provide information about the response of the systems which may be largely lost by "force-fitting" to a simplistic model. In particular, the drawdown at a given point in the system due to pumping at varying rates from a well can be predicted if the constant-rate drawdown curve at that point has been determined by test pumping of the well.

The use of constant-rate drawdown curves to predict the behavior of a ground-water system can be facilitated by introducing the concept of "specific drawdown curves," here defined as the constant-rate drawdown curves for a unit pumping rate. In equation form this definition can be expressed by:

$$X_{ij}(t) = \frac{s_{oij}(t)}{Q_{oj}} \qquad (12)$$

where $X_{ij}(t)$ = the specific drawdown curve for well i in response to pumping from well j, and $s_{oij}(t)$ is the constant-rate drawdown curve for well i due to pumping at a constant rate Q_{oj} from well j. The name "specific drawdown curve" is proposed here in analogy with the familiar term "specific capacity." The specific drawdown is the drawdown in an observation well for a unit pumping rate from a pumped well. It represents the response of the linear ground-water system at point i to a unit rate of withdrawal at point j. In general systems theory, the corresponding function goes by names such as "indicial admittance" (Wylie, 1960). In the unit hydrograph theory of surface-water hydrology, the corresponding function is the "S-curve" or

"S-hydrograph" (Dooge, 1973), and the problem discussed in this paper of determining $s_0(t)$, or $X_{ij}(t)$, if $s(t)$ and $Q(t)$ are measured, is equivalent to the identification problem of surface hydrology. The linear theory of surface hydrologic systems (Dooge, 1973) may therefore provide a useful source of theory and experience for the analysis of subsurface hydrologic systems.

With the foregoing definition of specific drawdown, equation (1) can be written in the form:

$$s_{ij}(t) = \sum_{k=0}^{n_j} (Q_{kj} - Q_{k-1,j}) X_{ij}(t - t_k) \qquad (13)$$

where $s_{ij}(t)$ is the drawdown in well i due to pumping from well j; Q_{kj} is the rate of pumping from well j during the k^{th} pumping interval starting at time $t = t_k$ ($Q_{kj} = 0$ for $k = -1$, $t_k = 0$ for $k = 0$); and n_j is the number of pumping steps for well j.

For continuously varying pumping rates, equation (13) can be written in integral form (e.g., Wylie, 1960, p. 338):

$$s_{ij}(t) = Q_{oj} X_{ij}(t) + \int_0^t \frac{dq_j(t')}{dt'} X_{ij}(t - t')\, dt' \qquad (14)$$

where Q_{oj} = initial rate of pumping from well j starting at time = 0; and $q_j(t)$ is the time-variable rate of pumping from well j.

The total drawdown at well i due to pumping from m wells is:

$$s_i(t) = \sum_{j=1}^{m} s_{ij}(t) \qquad (15)$$

Equations (13), (14), and (15) allow prediction of the drawdown at point i once the specific drawdown curves $X_{ij}(t)$ have been determined.

Specific drawdown curves can be calculated by the same methods as those for constant-rate drawdown curves described in this paper. The only additional step is to divide by the pumping rate [equation (12)] For some problems such as type-curve analysis, use of specific drawdown curves rather than constant-rate curves would only introduce an unnecessary extra calculation step. For problems such as prediction of drawdowns by means of equations (13), (14), or (15), the use of specific drawdown curves allows a more concise and clear statement of the problem. The essential point is that either constant-rate or specific drawdown curves provide a direct description of the response of the ground-water system to pumping.

The usefulness of equations (13), (14), and (15) can be much increased by use of the reciprocity principle (McKinley *et al.*, 1968). This principle states that for any pair of wells i and j, the constant-rate drawdown curve will be the same, whether well i is pumped and j is the observation well, or vice versa. In terms of specific drawdown curves, the reciprocity principle is expressed by:

$$X_{ij}(t) = X_{ji}(t) \qquad (16)$$

For example, by using the reciprocity principle, the interference between two production wells can be accurately predicted by pumping only one, measuring the drawdown in the other, and using this drawdown data to obtain the specific drawdown curve for the pair. Similarly, if it is necessary to predict the drawdown at a particular point due to pumping from a number of wells (e.g., for pressure relief), then it would suffice to pump a well at that point and observe the drawdown in all the proposed pumping wells. The specific drawdown curves obtained in this manner can then be used with equation (15) to predict the total drawdown at the point in question for any combination of pumping regimes at the other wells.

McKinley *et al.* (1968) show that the reciprocity principle holds for a heterogeneous medium, and they state that "the reservoir must not be pressure sensitive." In other words, the properties of the system must not change with changes of fluid pressure. It is not immediately clear from the proof of the reciprocity principle given by McKinley *et al.* whether it is in fact valid under the same general conditions as the principle of superposition, i.e. for any linear time-invariant ground-water flow system.

EXAMPLES

The following two examples of actual cases are presented mainly to illustrate the method of calculation of constant-rate drawdown curves. The practical usefulness of the method can only be evaluated by applying it for a variety of problems and hydrogeologic conditions. Such an evaluation is beyond the scope of this paper.

Three-Step Pumping Test Near Iuka, Illinois

Walton (1970, pp. 345-349) described a three-step pumping test on a well near Iuka, Illinois. Strictly speaking, a constant-rate drawdown curve should not be calculated for this test because a seepage face developed in the pumping well during the test and the system was therefore not time-invariant. However, Walton gives drawdown data for observation wells, and this case history therefore provides a realistic example of the calculation method. (A literature review indicates that during step drawdown tests on production wells, drawdowns in observation wells are usually not measured, or at least are not published).

For the test at Iuka the pumping well (No. 3) was initially pumped at 0.341 l/sec (5.4 gpm). The pumping rate was then increased to 0.587 l/sec (9.3 gpm) at about 120 minutes after pumping started, and increased again to 0.801 l/sec (12.7 gpm) at about 180 minutes. Pumping stopped at 250 minutes. The measured drawdowns, s, in the observation well (No. 2) are summarized in Table 1. Some of these values have been calculated by interpolation from the data given by Walton. The pumping rates and measured drawdowns are also presented graphically in Figure 2.

To calculate the constant-rate drawdowns for well No. 2, using equation (2), the following values are used:

$$Q_0 = 0.341 \text{ l/sec}$$
$$Q_1 = 0.587 \text{ l/sec}$$
$$Q_2 = 0.801 \text{ l/sec}$$
$$t_1 = 120 \text{ minutes}$$
$$t_2 = 180 \text{ minutes}$$
$$\Delta t = 10 \text{ minutes}$$

With these numbers, equation (2) becomes:

$$s_0(t) = s(t) - 0.721\, s_0(t-120) - 0.628\, s_0(t-180) \quad \ldots\ldots (17)$$

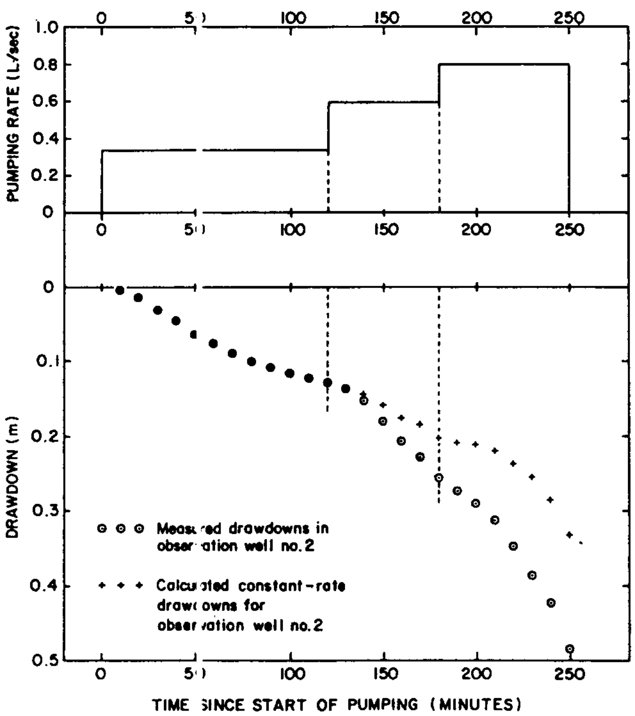

Fig. 2. Pumping rates, measured drawdowns, and calculated constant-rate drawdowns for a three-step pumping test at Iuka, Illinois (see Table 1).

Table 1. Measured Drawdowns and Calculated Constant-Rate Drawdowns for Well No. 2; Pumping Test at Iuka, Illinois (Field Data from Walton, 1970, p. 348) Data Points Indicated by Asterisks Are Interpolated from Data Given by Walton

t, minutes	*s* (m)	$s_0(t-120)$ (m)	$s_0(t-180)$ (m)	$s_0(t)$ (m)
0	0	0	0	0
10	.003*	0	0	.003
20	.014*	0	0	.014
30	.030*	0	0	.030
40	.043	0	0	.043
50	.064	0	0	.064
60	.076	0	0	.076
70	.088	0	0	.088
80	.098	0	0	.098
90	.107	0	0	.107
100	.116	0	0	.116
110	.122*	0	0	.122
120	.128	0	0	.128
130	.137	.003	0	.135
140	.152	.014	0	.142
150	.180	.030	0	.158
160	.207	.043	0	.176
170	.229	.064	0	.183
180	.256	.076	0	.201
190	.274	.088	.003	.209
200	.290	.098	.014	.211
210	.314	.107	.030	.218
220	.347	.116	.043	.236
230	.387	.122	.064	.259
240	.424	.128	.076	.284
250	.485	.135	.088	.332

* Interpolated data points.

The calculations are summarized in Table 1, and the calculated constant-rate drawdowns are shown in Figure 2.

As discussed by Walton, the sharp increase of drawdown in the observation well during the second and third step occurred because the water level in the pumping well declined below the top of the aquifer. This transient seepage face implies that the computed constant-rate drawdown curve is not reliable in this case because the system is not time-invariant; therefore, the principle of superposition is not applicable. The undulations of the calculated constant-rate curve (see Figure 2) are probably due to this time-variance of the system.

Drawdown data for the recovery phase are not available, else the computation of $s_0(t)$ could have been carried on beyond 250 minutes, with t_3 = 250 minutes and Q_3 = 0. It is likely that for this particular case, there would have been a marked discontinuity in the slope of the constant-rate drawdown curve at the transition from the pumping phase to the recovery phase of the test. Such a discontinuity would provide a direct indication that the conditions for applicability of the superposition principle were not satisfied.

Constant-Rate Pumping Test Near Estevan, Saskatchewan

Equation (10) was applied to drawdown and recovery data for a 41,520 minute (approximately 29 days) pumping test carried out in 1984, near Estevan, Saskatchewan (van der Kamp, 1985). Water levels were measured for the subsequent six months. The pumping well was completed in a confined buried-valley aquifer system described by Walton (1970, pp. 543-564). The hydrogeological setting of the aquifer is complex, and drawdown curves cannot be expected to follow any available theoretical type curves.

The measured drawdowns in observation well 11L-84 are shown in Figure 3 together with the extrapolated constant-rate drawdowns calculated by means of equation (10). The calculations of the extrapolated drawdowns are summarized in Table 2. For example, the constant-rate drawdown at $t = 3t_1 = 124{,}560$ minutes equals the sum of the measured drawdowns at $3t_1$, $2t_1$, and t_1:

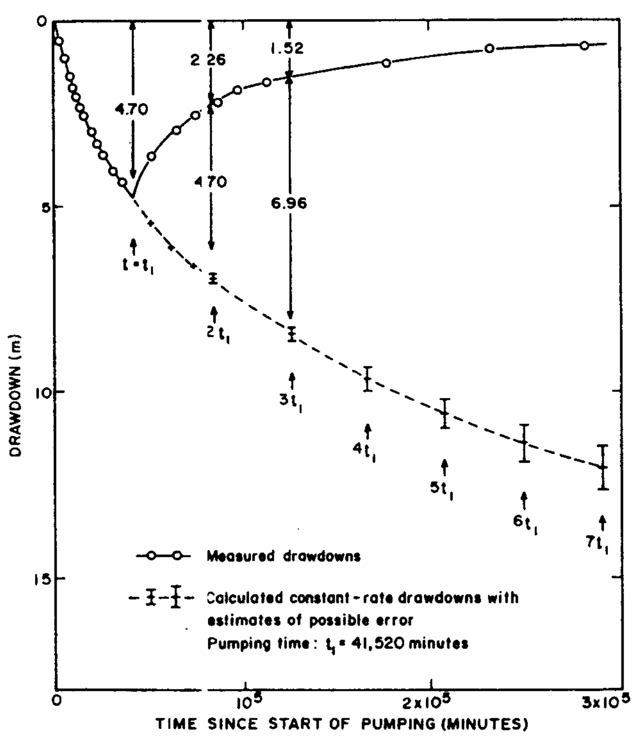

Fig. 3. Measured drawdowns and calculated constant-rate drawdowns for observation well 11L-84 during a pumping test near Estevan, Saskatchewan (see Table 2).

Table 2. Summary of Drawdown Data and Possible Error Data for Observation Well 11L-84 During Pumping Test Near Estevan, Saskatchewan, 1984-1985 [Duration of pumping was 41,520 minutes; n is the number of complete pumping periods that have elapsed (field data from van der Kamp, 1985)]

t, minutes	*n*	*s* (m)	*e* (m)	s_0 (m)	*E* (m)
0	0	0	0	0	0
10,380	0	1.75	.10	1.75	.10
20,760	0	3.02	.10	3.02	.10
31,140	0	4.00	.10	4.00	.10
41,520	1	4.70	.10	4.70	.10
51,900	1	3.65	.10	5.40	.20
62,280	1	3.07	.10	6.09	.20
72,660	1	2.60	.10	6.60	.20
83,040	2	2.26	.10	6.96	.20
124,560	3	1.52	.10	8.48	.30
166,080	4	1.18	.10	9.66	.40
207,600	5	.95	.10	10.61	.50
249,120	6	.76	.10	11.37	.60
290,640	7	.68	.10	12.05	.70

$s_0(3t_1) = 1.52 + 2.26 + 4.70 = 8.48$ meters (see Figure 3). The values of measured drawdowns tabulated in Table 2 were obtained by interpolation between the irregularly spaced data points, as indicated in Figure 3.

Long-term water-level records for the aquifer show that the static water level may vary by 15 to 20 cm during a year (Meneley *et al.*, 1979). Thus, the possible error in the values of drawdown can be taken to be ±0.10 m. The accumulated possible errors are shown by the error bars in Figure 3.

For the aquifer test data plotted in Figure 3, it can be seen that the use of recovery data lengthened the useful duration of the aquifer test from one month to more than six months. This was possible because significant drawdowns persisted for at least six months after pumping had stopped. (In this case, the long-lasting drawdown effect occurred because the aquifer is confined by a thick aquitard of very low permeability, which allows only minimal recharge to the aquifer.)

DISCUSSION

Limitations on Use of the Method

The method for calculating constant-rate drawdown curves which is described in this paper is only applicable if the ground-water system is linear and time-invariant. This is the major theoretical limitation on use of the method.

Linearity of the system means that the

ground-water flow must be governed by Darcy's law and that there must be a linear proportionality between change of hydraulic head and release of water from storage. Thus, the use of the method for drawdown and recovery in a pumped well will not be valid if non-Darcian (nonlinear) well losses are significant. Similarly, the method should not be used if the release of water from storage is not linearly proportional to head change or is different for falling head and for rising head as may, for instance, be the case for water-table aquifers.

Time-invariance of the system means that the geometry and hydraulic properties of the system must not change with time. Thus, for example, the method will give unreliable results if a significant part of the system is dewatered during the course of the test. Also the hydraulic conductivity and the storage coefficient should not be dependent on hydraulic head because the variation of head during pumping would then result in a corresponding time-variance of the system. For example, if the system includes pressure-sensitive fractures, hydraulic conductivity may vary considerably in time as the fluid pressure changes. Such a system would not satisfy the condition of time-invariance.

Time-invariance of the system also implies that the boundary conditions must remain constant or that changes of boundary conditions are explicitly taken into account. For example, the relative magnitude of the inflow at each point of the well screen should not vary during the course of the test. If a transient seepage face develops in the production well, then the system is not time-invariant, and the extrapolated constant-rate drawdowns may not be meaningful. The case of the three-step test near Iuka, Illinois, discussed in this paper, provides an example of this effect.

In practice, it may be difficult to judge whether a particular ground-water system can safely be treated as linear and time-invariant. Fortunately the method carries a built-in check: if the calculated constant-rate drawdown curves show marked undulations, or abrupt changes of slope associated with points in time when the pumping rate changed, then nonlinearity or time-variance of the system are indicated. This particular feature of the extrapolated constant-rate drawdown curves allows a sensitive check on the conditions of linearity and time-invariance which are implicit in many commonly used theoretical models. It may turn out that these assumptions are not as widely applicable as is commonly assumed.

The method can accommodate well-bore storage, but only if flow into the well and storage in the well are governed by linear laws. In practice, observation wells can usually be considered as part of the ground-water system, but production wells cannot be included because drawdowns in production wells are commonly influenced by nonlinear well losses. The expression "constant pumping rate" as used here normally refers to pumping from the formation, but may refer to pumping from a production well if well-bore storage is negligible or if nonlinear well losses are negligible. "Drawdown" refers to drawdown in the observation wells or drawdown in the formations, which may or may not be the same, depending on the response characteristics of the wells.

The method does not require a theoretical expression for the drawdown. Hence, it can accommodate irregular aquifer geometry, boundaries, partial penetration, anisotropy, and heterogeneity. It can accommodate fluids of different densities and viscosities within the system, but only if the fluid properties at any point in the system do not vary in time during the course of the pumping test. The method can also be applied for drawdowns in neighboring aquitards and aquifers.

The method is limited to stepped-rate tests. For the case of continuously varying pumping rates described by equation (14), the identification problem is much more difficult (Dooge, 1973). This difficulty can be avoided for ground-water systems by varying the pumping rate in discrete steps (a constant-head tank or similar devices can be used if pumping rate tends to vary with drawdown, e.g., Gale and Welhan, 1975).

The accumulation of errors is likely to be a major practical constraint on use of the proposed method. Special attention should therefore be paid to accurate measurement of the pumping rates, times of pumping-rate changes, water levels, and trends and fluctuations of the natural static water level. If possible, water-level measurements should be continued beyond the time when complete recovery is judged to have been attained, so as to obtain a reliable evaluation of trends in the static water level. Accurate measurement of pumping rates is particularly important if the method is to be used for a step-drawdown test because the calculated drawdown depends on the relative magnitude of the pumping rate for each step [see equation (2)]. For analysis of the recovery after a constant-rate test, high priority should be given to keeping the pumping rate constant; the absolute magnitude of the pumping rate is likely to be of lesser importance when it comes to interpretation of the data.

Applications of the Method

The most common and useful application of the method will probably be in the analysis of data for the recovery phase of constant-rate pumping tests. Other possible applications can be envisioned. It may, for instance, be possible to combine step-drawdown tests with aquifer performance tests. For such an application, the pumping rate would be varied in a small number of steps which together comprise the full duration of the test. The drawdown in the pumping well could then be analyzed for well efficiency, and the calculated constant-rate curves for the observation wells could be used to analyze the hydraulics of the ground-water system. Another potential application could be in cases where a constant-rate pumping test is inadvertently interrupted by a temporary break in pumping. If the beginning and end of the interruption are known, the test can be continued to its planned duration, and it should be possible to calculate a constant-rate drawdown curve with little loss of information.

As mentioned at the outset, recovery data can be analyzed by using available type curves to generate families of drawdown-recovery type curves. Such an approach was used by Ramey (1980) for analysis of pressure buildup tests and by Mishra and Chachadi (1985) for the analysis of flow to a large-diameter well. However, each individual drawdown type curve then leads to a family of drawdown-recovery type curves, and resulting multiplicity of type curves can become hard to oversee. The approach described here for analysis of recovery data is mathematically equivalent and may be easier to use.

Due to economic constraints, the duration of pumping for an aquifer test is frequently shorter than one would like. Analysis of recovery data by means of the method described here may allow one to obtain considerable additional information from an aquifer test. The essential point is that the duration of an aquifer test should not be considered to be equal to the duration of pumping, but should be taken to be the full length of time during which reliable drawdown measurements can be obtained.

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CJN 2022/04/23

DERIVATION OF THE van der KAMP (1989) METHOD

1. General formulation

The solution for the drawdown due to a time-varying discharge is given by:

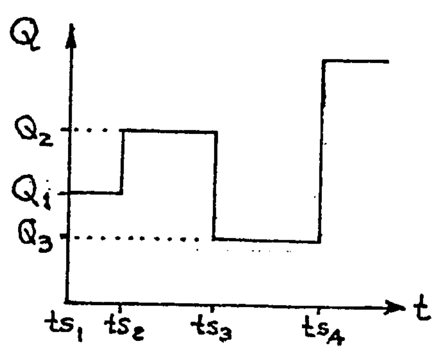
$$s(r,t) = \int_0^t Q(\tau)\, G(t-\tau)\, d\tau \qquad (1)$$

where:

$Q(t)$ = discharge history; and

$G(t)$ = Green's function for the particular conceptual model.

An arbitrary discharge history is represented as a set of discrete steps:



Using the Heaviside step function to represent the discretized discharge history:

$$Q(t) = \sum_{i=1}^{NP} \Delta Q_i\, H(t - ts_i) \qquad (2)$$

where:

$$\Delta Q_1 = Q_1$$
$$\Delta Q_2 = Q_2 - Q_1$$
$$\vdots$$
$$\Delta Q_{NP} = Q_{NP} - Q_{NP-1}$$

Substituting for the discrete discharge history in the general convolution integral yields:

$$s(r,t) = \int_0^t \sum_{i=1}^{NP} \Delta Q_i \, H(\tau - ts_i)\, G(t-\tau)\, d\tau \qquad (3$$

Re-arranging the order of integration and summation yields:

$$s(r,t) = \sum_{i=1}^{NP} \Delta Q_i \int_0^t H(\tau - ts_i)\, G(t-\tau)\, d\tau \qquad (4)$$

Now, making use of the definition of the Heaviside step function, the expression for $s(r,t)$ reduces to:

$$s(r,t) = \sum_{i=1}^{NP} \Delta Q_i \int_{ts_i}^{t} G(t-\tau)\, d\tau \qquad (5)$$

Defining a new variable of integration:

$$\xi = t - \tau \longrightarrow d\xi = -\,d\tau$$

$$\tau = ts_i \longrightarrow \xi = t - ts_i$$

$$\tau = t \longrightarrow \xi = 0$$

The integral reduces to:

$$s(r,t) = \sum_{i=1}^{NP} \Delta Q_i \int_0^{t-ts_i} G(\xi)\, d\xi \qquad (6)$$

The form of the solution presented by van der Kamp (1989) is derived by expanding the discretized form of the convolution integral:

$$s(r,t) = \Delta Q_1 \int_0^{t-ts_1} G(\xi)\,d\xi + \Delta Q_2 \int_0^{t-ts_2} G(\xi)\,d\xi + \Delta Q_3 \int_0^{t-ts_3} G(\xi)\,d\xi + \cdots \Delta Q_{NP} \int_0^{t-ts_{NP}} G(\xi)\,d\xi \quad \text{—(7)}$$

Now, recalling that:

$$\Delta Q_1 = Q_1 \qquad ; \quad ts_1 = 0$$

$$\Delta Q_2 = Q_2 - Q_1$$

$$\Delta Q_3 = Q_3 - Q_2$$

$$\vdots$$

$$\Delta Q_{NP} = Q_{NP} - Q_{NP-1}$$

the solution for the drawdown can be expanded as:

$$s(r,t) = Q_1 \int_0^{t} G(\xi)\,d\xi + (Q_2 - Q_1) \int_0^{t-ts_2} G(\xi)\,d\xi + (Q_3 - Q_e) \int_0^{t-ts_3} G(\xi)\,d\xi + \cdots (Q_{NP} - Q_{NP-1}) \int_0^{t-ts_{NP}} G(\xi)\,d\xi \quad \text{—(8)}$$

Making a change of terminology :

$$Q_1 \int_0^X G(\xi)\, d\xi = s_1(x) \qquad \text{———(9)}$$

the solution for the drawdown becomes :

$$s(r,t) = s_1(t) + \frac{(Q_2 - Q_1)}{Q_1} s_1(t - ts_2)$$

$$+ \frac{(Q_3 - Q_2)}{Q_1} s_1(t - ts_3) + \cdots \frac{(Q_{NP} - Q_{NP-1})}{Q_1} s_1(t - ts_{NP}) \qquad \text{———(10)}$$

→ THIS IS van der KAMP'S EQ.$^{\text{n}}$ (1).

Returning to summation notation, van der Kamp's EQ.$^{\text{n}}$ (1) becomes :

$$s(r,t) = s_1(t) + \sum_{i=2}^{NP} \frac{(Q_i - Q_{i-1})}{Q_1} s_1(t - ts_i)$$

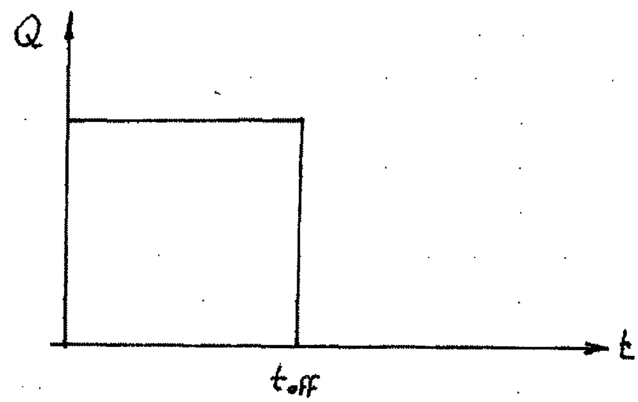
Solving for $s_1(t)$ yields :

$$s_1(t) = s(r,t) - \sum_{i=2}^{NP} \frac{(Q_i - Q_{i-1})}{Q_1} s_1(t - ts_i) \qquad \text{———(11)}$$

→ THIS IS van der KAMP'S EQ.$^{\text{n}}$ (2).

2. Check of the van der Kamp method

Let us check the van der Kamp method for the simple case of pumping at a constant rate followed by recovery.



$$ts_1 = 0 \quad , \quad Q_1 = Q$$
$$ts_2 = t_{off} \quad , \quad Q_2 = 0$$

- For $0 \le t \le t_{off}$, $s_1(t) = s(t)$
- For $t > t_{off}$, $s_1(t) = s(t) - \left[\frac{(0)-(Q)}{(Q)}\right] s_1(t-t_{off})$

$$= s(t) + s_1(t-t_{off})$$

From the principle of superposition, we know that the drawdown at any time after the end of pumping is:

$$s(t) = s_1(t) - s_1(t-t_{off}) \qquad \text{—}(*)$$

where $s_1(t)$ represents the drawdown that would have been observed if pumping had continued, and $s_1(t-t_{off})$ represents the effect of a well injecting water starting at $t = t_{off}$.

Rearranging $(*)$:

$$s_1(t) = s(t) + s_1(t-t_{off})$$

→ This is identical to the expression obtained by working through van der Kamp's theory. An example calculation is shown in the next figure.

A GENERAL METHOD FOR USING RECOVERY DATA FOR PUMPING TESTS IN COMPLEX HYDROGEOLOGICAL SETTINGS

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ABSTRACT
Collection of water-level recovery data is a common practice for pumping tests. The resulting data can provide some of the most useful information from the tests, but are rarely used to their full value. van der Kamp (1989) proposed a general method for the interpretation of recovery data that is easy to use and applicable for simple or complex hydrogeology, depending only on the principle of superposition. No other assumptions about the properties and geometry of the formations are required. The method can greatly increase the value of pumping tests by extending their effective duration for as long as significant residual drawdowns can be measured.

RÉSUMÉ
La collection de données de rétablissement de niveau d'eau est une pratique commune pour des essais de pompage. Les données en résultant peuvent fournir une grande partie des informations les plus utiles des essais, mais sont rarement employées à leur pleine valeur. van der Kamp (1989) a proposé une méthode générale pour l'interprétation des données de rétablissement qui est facile à utiliser et qui est applicable pour hydrogéologie simple ou complexe, dépendant seulement du principe de la superposition. Aucune autre assomption au sujet des propriétés et géométrie des formations sont exigées. La méthode peut considérablement augmenter la valeur des essais de pompages en prolongeant la durée efficace des essais, aussi long que des abaissements résiduels significatifs peuvent être mesurés.

1 INTRODUCTION

Guidance documents for conducting pumping tests typically require that water levels be monitored for a specified time following the end of pumping. In our experience, frequently nothing is done with the recovery data after they have been collected, plotted, and included in the appendix to a report. In most cases, the cursory treatment of recovery data represents a genuine loss. Recovery data frequently provide some of the most reliable information from pumping tests.

The traditional approach to interpreting recovery data has involved the application of the Theis model of aquifer response with the Cooper-Jacob approximation of the Theis well function (Cooper and Jacob, 1946). It assumes an ideal, confined aquifer of infinite extent, which is rarely encountered in practice, even approximately. The Cooper-Jacob straight-line analysis has a particularly simple implementation for recovery analysis:

$$s = \frac{Q}{4\pi T} 2.303 \log_{10}\left\{\frac{t}{t - t_{off}}\right\} \qquad [1]$$

In Equation [1], s is the drawdown, Q is the pumping rate (assumed constant during pumping), T is the transmissivity, t is the elapsed time since the start of pumping, and t_{off} is the duration of pumping. Equation [1] can be used directly to estimate the transmissivity from the slope of the semi-log plot. Apart from the assumption of an ideal confined aquifer, this approach essentially breaks the pumping test up into two independently analyzed portions, the pumping period and the recovery period. These may or may not give comparable results for the transmissivity of the aquifer, depending on how well the assumption of an ideal aquifer is met, even though they apply to the same well-aquifer system.

van der Kamp (1989) introduced a different approach for working with recovery data. The approach is based only on the principle of superposition and does not require other assumptions about the hydraulic properties and geometry of the aquifer and adjacent formations. The approach provides a straightforward and useful extension of existing methods. It allows consideration of the pumping and recovery periods together, essentially extending the effective duration of the pumping test to as long as measurable drawdown persists. Our experience suggests that van der Kamp's approach has been largely overlooked. As far as we are aware, it has not been implemented in any of the widely used interpretation packages. This is an important oversight and this note has been prepared in part to renew interest in this approach.

2 DEVELOPMENT OF THE GENERAL THEORY

For a general linear conceptual model, the drawdown $s(r,t)$ caused by pumping at a variable rate $Q(t)$ can be written as:

$$s(r,t)=\int_0^t Q(\tau)\,G(r,t-\tau)\,d\tau \qquad [2]$$

Equation [2] is a general statement of the principle of superposition, and is referred to as a convolution integral. The term $G(r,t)$ represents the drawdown at a distance r caused by pumping for an instant at time $t = 0$, and is frequently referred to as the Green's function for a particular problem. van der Kamp's method considers an arbitrary pumping history represented by a set of discrete steps, as shown in Figure 1.

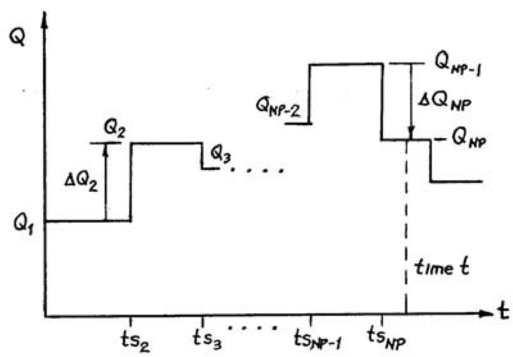

Figure 1. Discrete representation of an arbitrary pumping history

The *equivalent constant-rate drawdown*, s_1, is defined as the drawdown that would be observed at time t if the pumping rate had remained constant at a rate Q_1. For an arbitrary step pumping history, it follows from Equation [2] that the equivalent constant-rate drawdown is given by:

$$s_1(r,t)=s(r,t)-\left[\frac{(Q_2-Q_1)}{Q_1}s_1(r,t-ts_2)+\cdots+\frac{(Q_{NP}-Q_{NP-1})}{Q_1}s_1(r,t-ts_{NP})\right] \qquad [3]$$

In principle it is possible to reduce the drawdown data from any pumping test with varying pumping rates to the equivalent drawdown that would have been observed if the pumping rate had remained constant. This general principle depends only on the principle of superposition, and assumes only mathematical "linearity" of the equations that govern the flow. Linearity in turn means that the hydraulic properties of the formations do not change and that the boundary conditions remain constant (e.g., no dewatering of the formations).

3 APPLICATION FOR PUMPING AT A CONSTANT RATE FOLLOWED BY RECOVERY

Although the general form of the van der Kamp (1989) algorithm appears to be relatively complicated, it is particularly simple for the analysis of recovery following pumping at a constant rate. This is by far the most common pumping test practice. For this case, during the recovery period $NP = 2$, $t_{s2} = t_{off}$, and $Q_2 = 0$, and van der Kamp's general form reduces to:

$$s_1(r,t)=s(r,t)+s_1(r,t-t_{off}) \qquad [4]$$

This result can be interpreted directly: if pumping had continued, the drawdown at any time t would be equal to the actual drawdown at time t plus the drawdown observed at time t-t_{off}. Note that Equation [4] is not just limited to a recovery period that has the same duration as pumping. It can be applied for as long as the measured drawdown $s(r,t)$ is significant compared to the possible errors of measurement and uncertainties in what the water level would have been in the absence of pumping.

To illustrate the method, an idealized case of a well that penetrates the full thickness of an ideal confined aquifer is considered. The following parameter values are assumed: transmissivity, $T = 10^{-4}$ m^2/sec; storativity, $S = 10^{-4}$; pumping rate, $Q = 1.7\times10^{-3}$ m^3/sec; duration of pumping, t_{off} = 250 seconds; and radial distance, r = 10 m.

The calculated drawdown history is plotted in Figure 2.

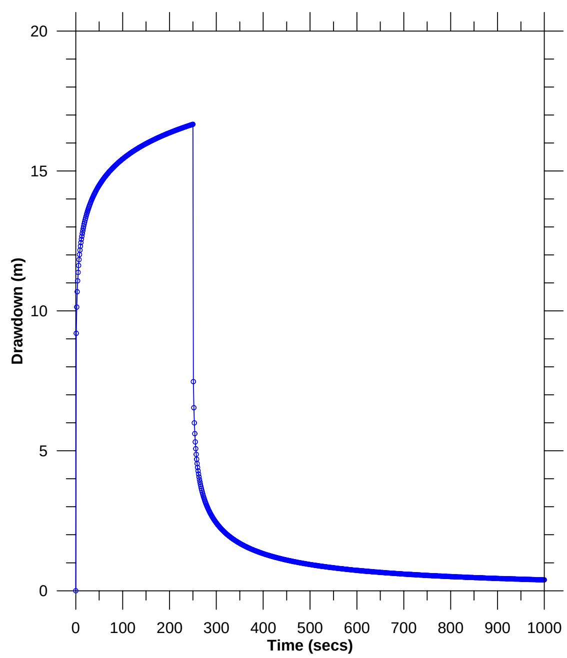

Figure 2. Calculated drawdown during pumping and recovery

To demonstrate the van der Kamp approach, the drawdown that would have been observed after 400 seconds if pumping had continued at a constant rate is calculated. The equivalent constant-rate drawdown at t = 400 seconds is given by:

$$s_1\left(t=400\text{ s}\right)=s\left(t=400\text{ s}\right)+s_1\left(t-t_{off}=150\text{ s}\right) \qquad [5]$$

At t = 400 seconds, the observed drawdown plotted in Figure 2 is 1.33 m. At t = 150 seconds, the well is still pumping; therefore $s_1 = s$ and the drawdown estimated from Figure 2 is $s_1(t = 150\text{ s})$ = 15.98 m. The equivalent drawdown is therefore:

$$s_1\left(t=400\text{ s}\right)=1.33\text{ m}+15.98\text{ m}=17.31\text{ m} \qquad [6]$$

The calculation is illustrated in Figure 3.

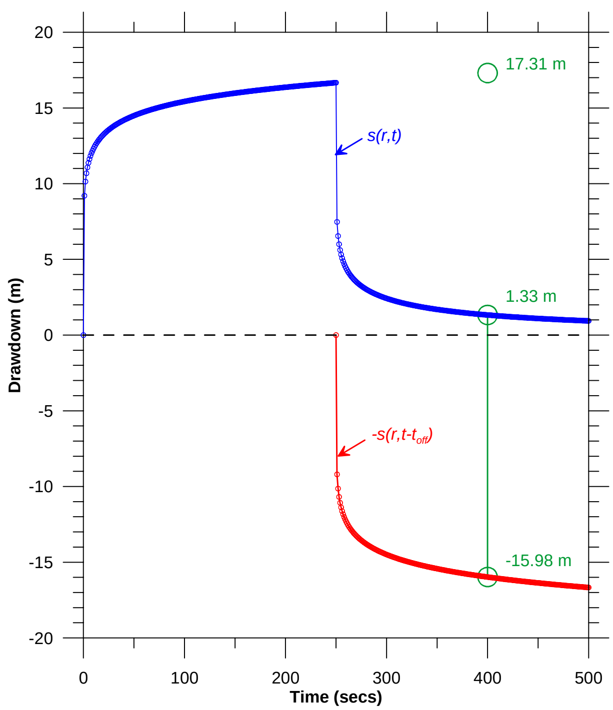

Figure 3. Calculation of equivalent constant-rate drawdown at 400 seconds

The results of applying the van der Kamp method for all of the results of the example are plotted in Figure 4.

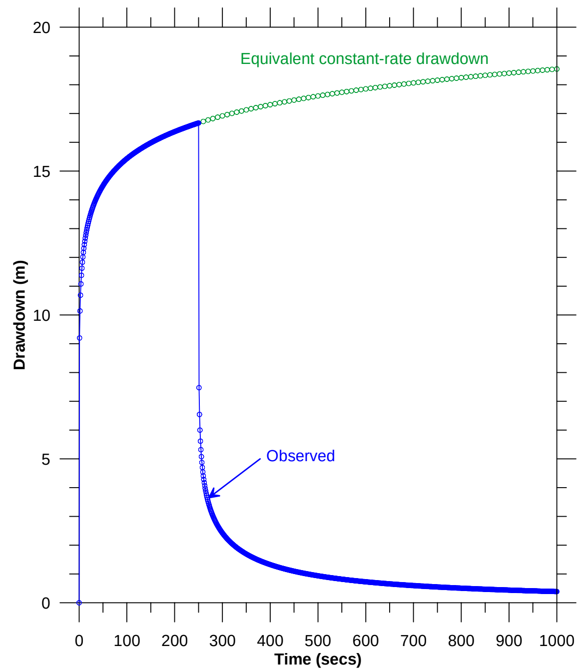

Figure 4. Actual drawdown and equivalent constant-rate drawdown

A simple but widely applicable illustration of the potential utility of the method can be drawn from the above example. Consider a pumping test with recovery data taken for the same time after pumping as the duration of pumping. The standard 24-hour test with 24 hours of recovery is a case in point. The residual drawdown after 24 hours of recovery is equal to the additional drawdown that would have occurred between 24 and 48 hours if pumping had continued. The one data point obtained after 24 hours of recovery already doubles the effective length of the pumping test, especially if further analysis is based on methods making use of semi-log or log-log plots of drawdown versus time. Numerous “24-24” pumping test analyses could make good use of this simplest of calculations. Other data points can also be calculated, as illustrated in Figures 3 and 4.

An additional advantage of making full use of the recovery data is that “noise” introduced into the drawdown data by irregularities of the pumping rate is much reduced during the recovery phase.

4 CASE STUDY

The utility of the van der Kamp approach is demonstrated by using the recovery data to extend the effective duration of a pumping test conducted in a confined buried-channel aquifer near Estevan, Saskatchewan. The test was conducted in 1984 and was reported in van der Kamp (1985; 1989). The aquifer is described in Walton (1970), van der Kamp and Maathuis (2002), and Maathuis and van der Kamp (2003). This is a complex semi-confined channel aquifer, involving complicating factors such as several intersecting channels, partial blockages, lateral inflow from surrounding formations and unknown regional permeability of the overlying glacial till aquitard. No simple analytical aquifer model could be expected to apply and the numerical model that was developed was highly unconstrained.

The aquifer was pumped at a constant rate for 41,520 minutes (about 29 days), and water levels following the end of pumping were monitored for an additional 249,000 minutes (173 days). Drawdowns at observation well 11L-84 during the pumping and recovery periods are shown in Figure 5 (data from Figure 3 of van der Kamp, 1989). For subsequent analysis, the original observations are supplemented with interpolated values indicated by the crosses. The interpolated drawdown observations, taken from van der Kamp (1989; Table 2), are smoothed slightly with respect to the original observations.

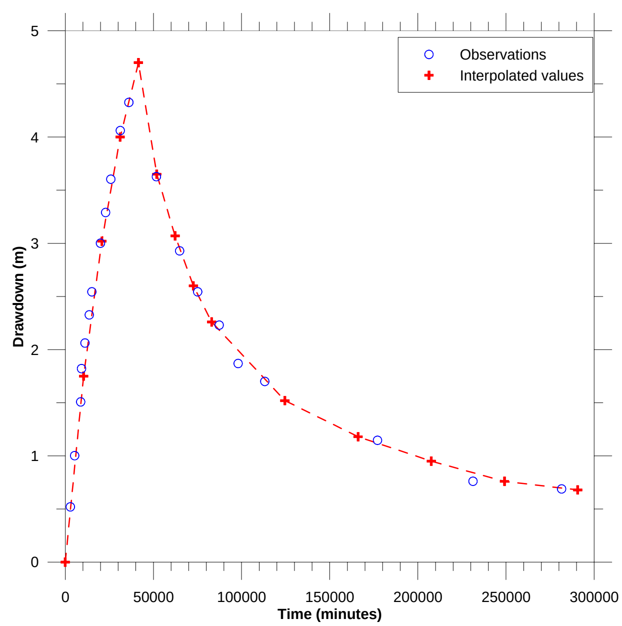

Figure 5. Raw drawdown data

Complete results obtained from applying van der Kamp's method are shown in Figure 6.

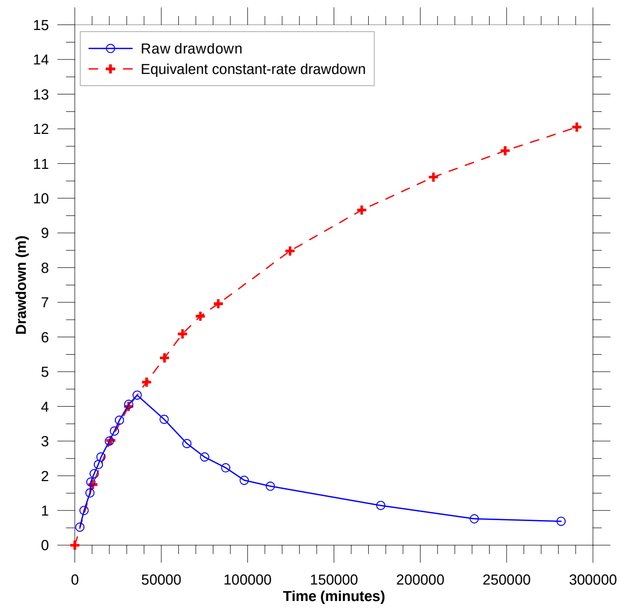

Figure 6. Equivalent constant-rate drawdowns

In this example, the use of recovery data lengthens the useful duration of the pumping test from one month to more than six months. The implications of this extension are best illustrated by plotting the raw drawdowns and the equivalent constant-rate drawdowns against the logarithm of time, as shown in Figure 7.

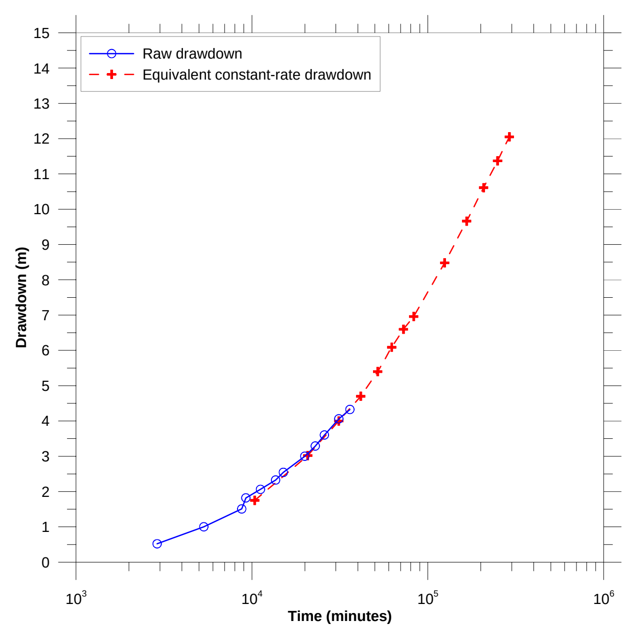

Figure 7. Semi-log plot raw and equivalent drawdown data

As shown in Figure 7, even after 29 days of pumping it is only possible to identify the beginning of the long-term trend of the drawdown. In contrast, the accelerating trend that is characteristic of a buried-channel aquifer is clearly evident in the equivalent constant-rate drawdowns. The drawdown at the end of pumping is 4.70 m. The equivalent constant-rate drawdown for the last recorded water level is 12.05 m.

The application of the van der Kamp analysis in this example is possible because significant drawdowns persisted more than six months beyond the end of pumping. The persistent drawdown reflects the conditions of the aquifer: the buried-channel aquifer is overlain by a thick aquitard of low conductivity, which allows only minimal recharge to the aquifer.

Subsequently the aquifer was pumped at a high rate for 6 years to supply cooling water for a coal-fired power plant (Maathuis and van der Kamp, 2003). The long-term drawdown due to such pumping was predicted on the basis of the 6 months of extrapolated drawdown illustrated in Figure 7, and the actual measured drawdown agreed closely with the prediction.

6 DISCUSSION

Robust and inexpensive pressure transducers have become widely available in recent years. These can be left securely in observation wells without requiring the continuous on-site presence of field staff. The collection of extended recovery data has therefore become easier and more economical. It may become standard practice to record water level data after the cessation of pumping for as long as it takes to attain full recovery. Such long-term monitoring of recovery has the additional advantage that it may allow a more robust estimate of changes of the "static" water level during the pumping and recovery period.

The authors' experience with pumping tests suggests that the general method for the analysis of recovery data described in van der Kamp (1989) could have enhanced the value of almost every pumping test that they have encountered, with only minor additional effort in data analysis. Full recognition and exploitation of the potential value of recovery data is therefore recommended to all practitioners.

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groundwater

Methods Note/

Using Recovery Data to Extend the Effective Duration of Pumping Tests

by Christopher J. Neville[1] and Garth van der Kamp[2]

Abstract

Collection of water-level recovery data is a common practice for pumping tests. The resulting data can provide some of the most useful information from the tests, but are rarely used to their full value. van der Kamp (1989) proposed a general method that uses recovery data to extend the effective duration of pumping. The method is straightforward to implement and applicable for simple or complex hydrogeologic settings. The only assumption invoked is that the response remains linear such that the principle of superposition can be applied. No other assumptions about the properties of the aquifer are required. The method can greatly increase the value of pumping tests by extending the effective duration of the tests for as long as significant residual drawdowns are observed.

Introduction

Guidance documents for conducting pumping tests typically require that water levels be monitored for a specified time following the end of pumping. In our experience, frequently nothing is done with the recovery data after they have been collected, plotted, and included in the appendix to a report. In most cases, the cursory treatment of recovery data represents a genuine loss. Recovery data frequently provide some of the most valuable information from pumping tests.

The traditional approach to interpreting recovery data has involved the application of the Theis model of aquifer response with the Cooper-Jacob approximation of the Theis well function (Theis 1935; Cooper and Jacob 1946). The aquifer is assumed to be confined, homogeneous and isotropic and of infinite extent. This approach essentially breaks the pumping test up into two independently analyzed portions, the pumping period and the recovery period. The separate analyses may or may not give comparable results for the transmissivity of the aquifer, depending on how well the assumption of an ideal aquifer is met, even though they are applied to the same well-aquifer system.

van der Kamp (1989) introduced a different approach for working with recovery data, which makes use of the principle of superposition and does not require other assumptions about the hydraulic properties and geometry of the aquifer and adjacent formations. The approach provides a straightforward and useful extension of existing methods. It allows consideration of the pumping and recovery periods together, extending the effective duration of the pumping test to as long as measurable drawdown persists. Our experience suggests that van der Kamp's approach has been largely overlooked. As far as we are aware it has not been implemented in any of the widely used interpretation packages. In our opinion this is an important oversight, and this note has been prepared to renew interest in this approach.

Method and Application

van der Kamp (1989) defined an *equivalent constant-rate drawdown*, s_1, as the drawdown that would be

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observed at time t if the pumping rate had remained constant at its initial rate Q_1. van der Kamp's method considers an arbitrary pumping history represented by a set of discrete steps. For the case of pumping at a constant rate followed by recovery, van der Kamp's general form reduces to

$$s_1(x_i, t) = s(x_i, t) + s_1(x_i, t - t_{\text{off}}) \qquad (1)$$

where $s(x_i, t)$ is the observed drawdown at the observation well, x_i are the coordinates of the observation well, t the total elapsed time since the start of pumping, and t_{off} the duration of pumping. This result has a direct physical interpretation: if pumping had continued, the drawdown at any time t would be equal to the drawdown observed at time t plus the drawdown observed at time $t - t_{\text{off}}$. Equation 1 is not limited to a recovery period equal to the duration of pumping, but can be applied for as long as the observed drawdown $s(r,t)$ is significant compared to the possible errors of measurement and uncertainties in what the water level would have been in the absence of pumping.

The application and utility of van der Kamp's approach is demonstrated by making use of the recovery data from a pumping test conducted in a confined buried-channel aquifer near Estevan, Saskatchewan. This is a complex semi-confined channel aquifer, involving complicating factors such as several intersecting channels, partial blockages, lateral inflow from surrounding formations and an unknown regional permeability of the overlying glacial till aquitard (Walton 1970; van der Kamp and Maathuis 2011). The test was conducted in 1984 and was reported in van der Kamp (1985, 1989). The aquifer was pumped at a constant rate of 0.076 m^3/s for 41,520 min (about 29 d), and water levels were monitored for an additional 249,000 min (173 d). Drawdowns at observation well 11L-84 during the pumping and recovery periods are shown in Figure 1. The original observations are supplemented with smoothed interpolated values indicated by the crosses (values are taken from Table 2 of van der Kamp 1989).

The calculation of the equivalent drawdown at any time beyond the end of pumping is straightforward. For example, after 83,040 min, the observed drawdown was 2.26 m. The drawdown at the end of pumping, $t = 41{,}520$ min, was 4.70 m. Therefore, if pumping had continued, the drawdown that would have been observed if pumping had been twice as long is

$$\begin{aligned} s_1(t = 83{,}040\,\text{min}) &= s(t = 83{,}040\,\text{min}) \\ &\quad + s_1(t = 83{,}040\text{ min} - 41{,}520\,\text{min}) \\ &= 2.26\text{ m} + 4.70\text{ m} = 6.96\text{ m}. \end{aligned}$$

The method can be applied beyond a time corresponding to twice the original duration of pumping, as long as there are observed drawdowns. In this example, beyond 41,520 min, use is made of the calculated equivalent constant-rate drawdowns. For example, after 124,560 min

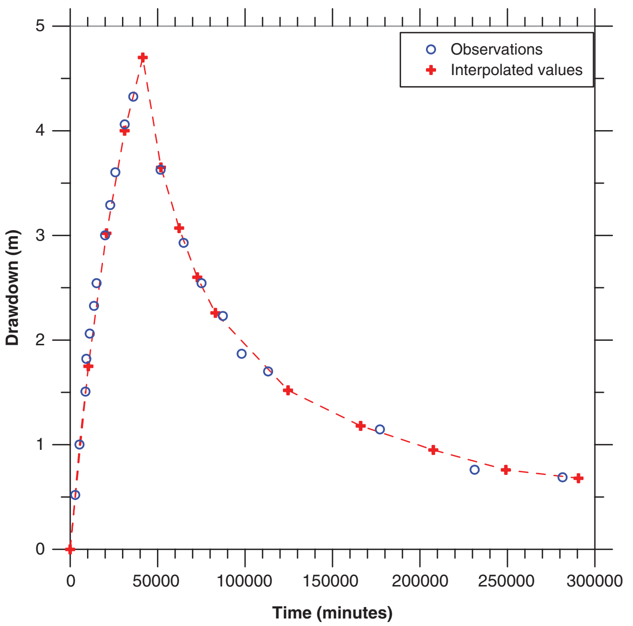

Figure 1. Drawdowns at observation well 11L-84 during the pumping and recovery periods.

the observed drawdown was 1.52 m. This corresponds to 83,040 min beyond the end of pumping. The equivalent constant-rate drawdown after 83,040 min calculated above is 6.96 m. Therefore, the constant-rate drawdown after 124,560 min is

$$\begin{aligned} s_1(t = 124{,}560\,\text{min}) &= s(t = 124{,}560\,\text{min}) \\ &\quad + s_1(t = 124{,}560\text{ min} - 41{,}520\,\text{min}) \\ &= 1.52\text{ m} + 6.96\text{ m} = 8.48\text{ m}. \end{aligned}$$

Complete results obtained by applying van der Kamp's method are shown in Figure 2.

Insights from Application of the van der Kamp Method

In the Estevan example, the use of recovery data to calculate equivalent constant-rate drawdowns lengthens the effective duration of the pumping test from 1 month to more than 6 months. The implications of this extension are best illustrated by plotting the drawdowns for multiple observation wells. As shown in Figure 3, even after 29 d of pumping it is only possible to identify the beginning of the long-term trends in the drawdown records. In contrast, the extended drawdown records plotted in Figure 4 show clear increasing trends that are characteristic of buried-channel aquifers. The dashed lines plotted in Figure 4 represent a match to the data with the Theis solution with $T = 0.023$ m^2/s and $S = 10^{-4}$, assuming parallel impermeable valley walls 400 m on either side of the pumping well. The application of the van der Kamp analysis in this example is possible because significant drawdowns persist more than 6 months beyond the end of pumping. The equivalent drawdowns provide insights

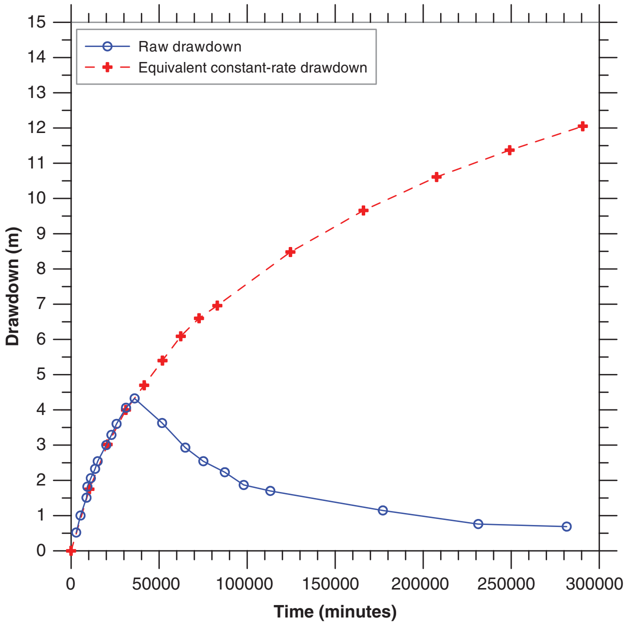

Figure 2. Complete results obtained by applying van der Kamp's method.

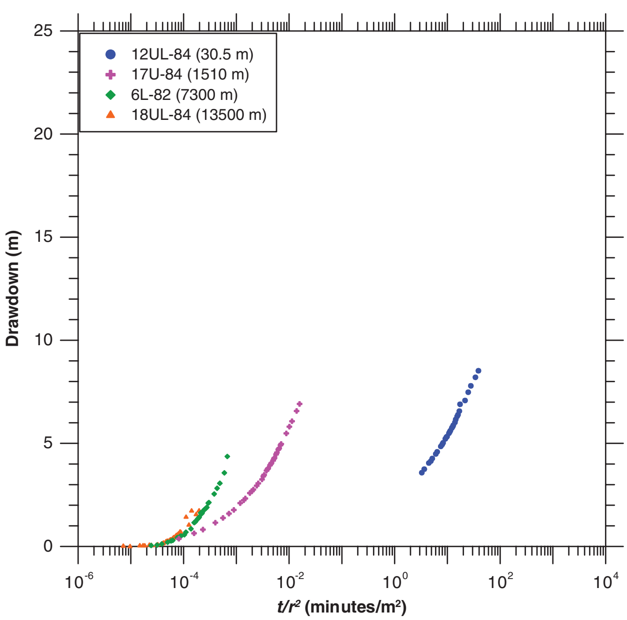

Figure 3. Composite plot of drawdowns for four observation wells.

for the diagnosis of the aquifer system that are not obvious from the drawdown records.

Conclusion

The authors' experience with pumping tests suggests that application of the general method for the treatment of recovery data described by van der Kamp (1989) could have enhanced the value of almost every pumping test that they have encountered, with only minor additional effort in data analysis.

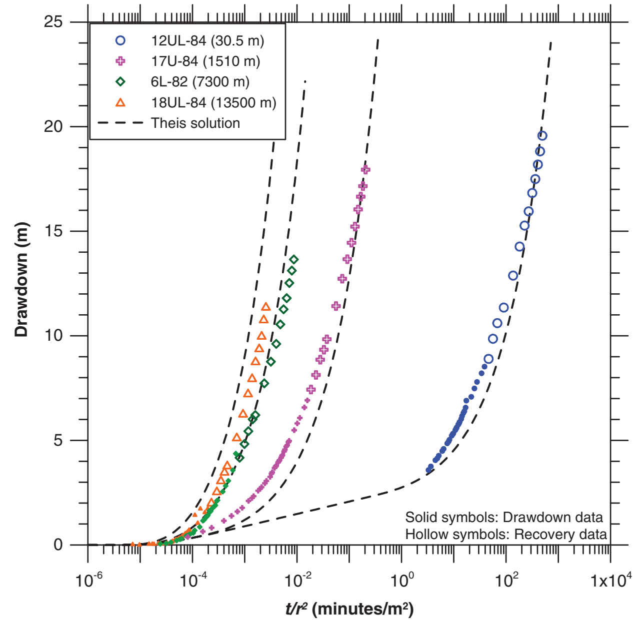

Figure 4. Composite plot of equivalent constant-rate drawdowns, with Theis solution for a strip aquifer.

Compact, robust, and inexpensive pressure transducers are now widely available. These can be left securely in observation wells without requiring the continuous on-site presence of field staff. The collection of extended recovery data has therefore become much easier and more economical. It may become standard practice to record water-level data after the cessation of pumping for as long as it takes to attain full recovery. Just one application of recovery data is described in this note. Long-term monitoring of recovery may allow a more robust estimate of changes of the ambient water level during the pumping and recovery period. Making full use of recovery data reduces the significance of the "noise" introduced into the drawdown data by irregularities in the pumping history. Recovery data are also important for identifying processes that may give rise to changes in water levels during a pumping test that are not caused by pumping, including the effects of nearby pumping, fluctuations in barometric-pressure and earth tides, and can be used to check the calibration of the pressure transducers. Full recognition and exploitation of the potential value of recovery data are recommended to all practitioners.

Acknowledgment

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